2.2 Differentiation Rules

Recall the power rule.... $f(x)=x^2$

$f'(x)=2x$

What happens when we try to take the derivative of $f(x)=3x^2$?

$f(x)=cg(x)$

Proof:
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{cg(x+h)-cg(x)}{h}$$

$$=c\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$=cg'(x)$$

The Constant Multiple Rule:
If $f(x) = cg(x)$ for any constant c, then $f'(x)= cg'(x)$

Ex.1 Find the derivative

a) $2x^{-2}$

$f'(x) = -4x^{-3}$

$= -\frac{4}{x^3}$

b) $-3\sqrt[3]{x}$

$f(x) = -3x^{\frac{1}{3}}$

$f'(x) = -x^{-\frac{2}{3}}$

$= \frac{-1}{x^{\frac{2}{3}}}$

Investigate the sum and difference rules.....

Proof:

Given f(x)=p(x) + q(x)

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{[p(x+h)+q(x+h)]-[p(x)+q(x)]}{h}$$

$$= \lim_{h\to 0}\left\{\frac{[p(x+h)-p(x)]}{h}+\frac{[q(x+h)-q(x)]}{h}\right\}$$

$$= \lim_{h\to 0}\left\{\frac{[p(x+h)-p(x)]}{h}\right\}+\lim_{h\to 0}\left\{\frac{[q(x+h)-q(x)]}{h}\right\}$$

$$= p'(x)+q'(x)$$

If functions p(x) and q(x) are differentiable, and $f(x)=p(x)\pm q(x)$, then $f'(x)=p'(x)\pm q(x)$.

Ex.2 Find the derivative

a) $y=-2x^4-3x^2+5$

$y' = -8x^3 - 6x$

b) $s=t^2(t-1)$

$= t^3 - t^2$

$s' = 3t^2 - 2t$

c) $f(x)=(2x-3)(x+1)$

$= 2x^2 - x - 3$

$f'(x) = 4x - 1$

d) $f(x)=\sqrt[3]{27x^6}$

$= 3\sqrt[3]{x^6}$

$= 3x^2$

$f'(x) = 6x$

$\left\{ (x^6)^{\frac{1}{3}} = x^{6 \cdot \frac{1}{3}} \right.$

Ex.3 Find the equation of the tangent of $y=\frac{\sqrt{x}-2}{\sqrt[3]{x}}$ at the point (1,-1)

$y = (x^{\frac{1}{2}} - 2) \cdot x^{-\frac{1}{3}}$

$= x^{\frac{1}{6}} - 2x^{-\frac{1}{3}}$

$\boxed{y' = \frac{1}{6}x^{-\frac{5}{6}} + \frac{2}{3}x^{-\frac{4}{3}}}$

$y = \frac{x^{\frac{1}{2}} - 2}{x^{\frac{1}{3}}}$

$= \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} - \frac{2}{x^{\frac{1}{3}}}$

$= x^{\frac{1}{6}} - 2x^{-\frac{1}{3}}$

② Sub $x=1$

$y' = \frac{1}{6}(1)^{-\frac{5}{6}} + \frac{2}{3}(1)^{-\frac{4}{3}}$

$= \frac{1}{6} + \frac{2}{3}$

$\boxed{m = \frac{5}{6}}$

③ Sub $m=\frac{5}{6}$ and $(1,-1)$

$y = mx + b$

$-1 = \frac{5}{6}(1) + b$

$b = -\frac{11}{6}$

$\therefore y = \frac{5}{6}x - \frac{11}{6}$

HMK
Pg. 82 # 3df,4cef,
5c, 6b,8d,9df,14,
16, 18, 25b,~~26~~