2.5 Chain Rule

The Chain Rule:

If f and g functions whose derivatives exist and $h(x)=f[g(x)]$, then:

$$h'(x)=f'[g(x)]\ g'(x)$$

ex: $f(x) = 3(☺)^5$

$f'(x) = 15(☺)^4 \frac{d}{dx}(☺)$

ex: $f(x)=(2x^2)^3$

$f'(x)=3(2x^2)^2(4x)$

☆ In Leibniz notation:

If y is a function of u and u is a function of x, then:

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Ex.1 Find the derivative of $f(x)=(2x^2)^3$ using the following methods.

a) simplifying and using the power rule

b) using the chain rule

Is there a difference????

$(2x^2)^3 = 2^3x^6$

a) $f(x) = 8x^6$

$f'(x) = 48x^5$

b) $f'(x) = 3(2x^2)^2 \cdot 4x$

$= 12x \cdot 4x^4$

$= 48x^5$

Ex. 2 Determine f'(x). Do not simplify, but write with no negative exponents.

a) $f(x)=(3x^2-7x+4)^3$

$f'(x) = 3(3x^2-7x+4)^2(6x-7)$

b) $f(x)=\sqrt[3]{x^2 - 5}$

$= (x^2-5)^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}(x^2-5)^{-\frac{2}{3}}(2x)$

$= \frac{1}{3}\ \frac{1}{(x^2-5)^{\frac{2}{3}}}\ \frac{2x}{1}$

$= \frac{2x}{3(x^2-5)^{\frac{2}{3}}}$

c) $f(x)= \frac{5}{(2x^2 - 7x + 1)^3}$

$= 5(2x^2-7x+1)^{-3}$

$f'(x) = -15(2x^2-7x+1)^{-4} \cdot (4x-7)$

$= \frac{-15(4x-7)}{(2x^2-7x+1)^4}$

d) $f(x)= (3x^2- 5x - 1)^{\frac{5}{3}}$

$f'(x) = \frac{5}{3}(3x^2-5x-1)^{\frac{2}{3}}(6x-5)$

Ex. 3 Determine f'(x). Simplify your answer by factoring.

a) $f(x) = (3x^2 + 1)^4 (2 - 3x)^3$

Product Rule

(and use chain rule)

$f'g + fg'$

$$f'(x) = \overbrace{4(3x^2+1)^3(6x)}^{\text{CHAIN RULE}}(2-3x)^3 + (3x^2+1)^4 \underbrace{(3)(2-3x)^2(-3)}_{\text{CHAIN RULE}}$$

$$= 3(3x^2+1)^3(2-3x)^2\left[8x(2-3x) - 3(3x^2+1)\right]$$

$$= 3(3x^2+1)^3(2-3x)^2(16x - 24x^2 - 9x^2 - 3)$$

$$= 3(3x^2+1)^3(2-3x)^2(-33x^2 + 16x - 3)$$

b) $f(x) = \dfrac{(2x^2 - 5)^6}{(3x-1)^7}$

$\dfrac{f'g - fg'}{g^2}$

QUOTIENT RULE!

and chain rule

$$f'(x) = \frac{6(2x^2-5)^5(4x)(3x-1)^7 - (2x^2-5)^6(7)(3x-1)^6(3)}{\left[(3x-1)^7\right]^2}$$

$$= \frac{3(2x^2-5)^5\cancel{(3x-1)^6}\left[8x(3x-1) - 7(2x^2-5)\right]}{(3x-1)^{\cancel{14}\,8}}$$

$$= \frac{3(2x^2-5)^5(24x^2 - 8x - 14x^2 + 35)}{(3x-1)^8}$$

$$= \frac{3(2x^2-5)^5(10x^2 - 8x + 35)}{(3x-1)^8}$$

Ex. 4 Use Leibniz notation to find $\frac{dy}{dx}$ at the given value of x.

$y = 4u^2 - 3u, \quad u = \sqrt{x^3+1}, \quad x = 2$

If we sub back in, same as

$y = 4\left(\sqrt{x^3+1}\right)^2 - 3\left(\sqrt{x^3+1}\right)$

But chain rule helps us here.

$\frac{dy}{dx} = \frac{dy}{\cancel{du}} \cdot \frac{\cancel{du}}{dx}$

$y = 4u^2 - 3u$

$\frac{dy}{du} = 8u - 3$

$u = \sqrt{x^3+1}$ $(x^3+1)^{\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2}(x^3+1)^{-\frac{1}{2}}(3x^2)$

$= \frac{3x^2}{2(x^3+1)^{\frac{1}{2}}}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= (8u-3)\left(\frac{3x^2}{2(x^3+1)^{\frac{1}{2}}}\right)$

if $x=2$
$u = ?$

$u = \sqrt{x^3+1}$
$= \sqrt{9}$
$= 3$

Sub in $x = 2$ and $u = 3$

$\left.\frac{dy}{dx}\right|_{x=2} = \left[8(3)-3\right]\left[\frac{3(2)^2}{2(2^3+1)^{\frac{1}{2}}}\right]$

$= 21\left(\frac{12}{6}\right)$

$\boxed{= 42}$

Ex. 5 Determine the equation of the tangent to the curve $y = \dfrac{1}{\sqrt{20 - x^4}}$ at the point $(2, \frac{1}{2})$.

① Find derivative
② Sub $x = 2$ to find slope at $(2, \frac{1}{2})$
③ Use $y = mx + b$ to build eq$\underline{n}$

$= (20 - x^4)^{-\frac{1}{2}}$

①
$y' = -\frac{1}{2}(20 - x^4)^{-\frac{3}{2}}(-4x^3)$

② sub in $x = 2$

$y' = -\frac{1}{2}(20 - 2^4)^{-\frac{3}{2}}(-4(2)^3)$

$= -\frac{1}{2}\underbrace{(4)^{-\frac{3}{2}}}(-32) \quad \left(\left[(4)^{\frac{1}{2}}\right]^3\right)^{-1}$

$= 16\left(\frac{1}{8}\right)$

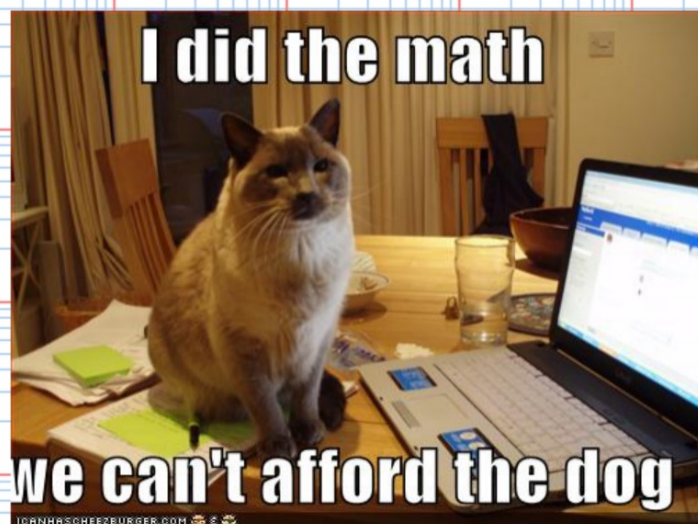
$= 2$

③ sub in $m = 2$ into $y = mx + b$
and $(2, \frac{1}{2})$

$\frac{1}{2} = 2(2) + b$
$b = -\frac{7}{2}$

$\therefore y = 2x - \frac{7}{2}$

Hmk Pg. 105
#4cef, 5ef, 7,8acf,
10,12,13bc, 16,17



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