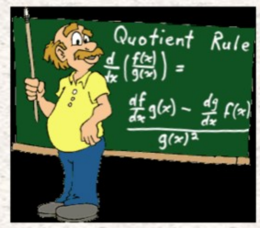
2.4 The Quotient Rule

The Quotient Rule:

If $h(x) = \dfrac{f(x)}{g(x)}$, then $h'(x) = \dfrac{f'(x)\,g(x) - f(x)\,g'(x)}{[g(x)]^2}$

In Leibniz notation: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{\dfrac{du}{dx}v - u\dfrac{dv}{dx}}{v^2}$

Proof (using product rule):



if $h(x) = \dfrac{f(x)}{g(x)}$, then $h(x)g(x) = f(x)$

Use product rule to take derivative of f(x)

$$h'(x)g(x) + h(x)g'(x) = f'(x)$$

Solve for h'(x)

$$h'(x) = \frac{f'(x) - h(x)\,g'(x)}{g(x)}$$

Substitute $h(x) = \dfrac{f(x)}{g(x)}$

$$h'(x) = \frac{f'(x) - \dfrac{f(x)}{g(x)}\,g'(x)}{g(x)}$$

Simplify (multiply by $\dfrac{g(x)}{g(x)}$)

$$h'(x) = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{[g(x)]^2}$$

Eg. 1 Find the derivative. Simplify.

a) $f(x)= \dfrac{2x^2 - 3x + 1}{x^2 + 3}$ $\quad \dfrac{f}{g}$

$$f'(x) = \frac{f'g - fg'}{g^2}$$

$$= \frac{(4x-3)(x^2+3) - (2x^2-3x+1)(2x)}{(x^2+3)^2}$$

$$= \frac{\cancel{4x^3}+12x-3x^2-9 - (\cancel{4x^3}-6x^2+2x)}{x^4+6x^2+9}$$

$$= \frac{3x^2+10x-9}{x^4+6x^2+9}$$

What to do?
...multiply out numerator
OR use product rule???

Product rule here would be more work! NO THANKS!

b) $g(x)= \dfrac{(2x+1)(x-3)}{2 - x}$

$$= \frac{2x^2-5x-3}{2-x} \quad \frac{f}{g}$$

$$g'(x) = \frac{f'g - fg'}{g^2}$$

$$= \frac{(4x-5)(2-x) - (2x^2-5x-3)(-1)}{(2-x)^2}$$

$$= \frac{8x-4x^2-10+\cancel{5x}+2x^2-\cancel{5x}-3}{4-4x+x^2}$$

$$= \frac{-2x^2+8x-13}{x^2-4x+4}$$

Eg. 2 Determine the slope of the tangent to h(x) at x=2.

$$h(x)= \frac{5x + 2}{x - 4} \quad \frac{f}{g}$$

Same idea
1. Find $h'(x)$
2. Find $h'(2)$

$$h'(x) = \frac{f'g - fg'}{g^2}$$

$$= \frac{5(x-4) - (5x+2)(1)}{(x-4)^2}$$

$$= \frac{\cancel{5x} - 20 - \cancel{5x} - 2}{x^2 - 8x + 16}$$

$$= \frac{-22}{x^2 - 8x + 16}$$

$$h'(2) = \frac{-22}{4 - 16 + 16}$$

$$= -\frac{11}{2}$$

$\therefore m_{tan} = \frac{-11}{2}$

Eg. 3 Determine the point(s) on the curve y= $\frac{x}{x-1}$ where the tangent line is parallel to the line x + 4y = 1.

$y = \frac{x}{x-1}$ $\frac{f}{g}$

① find y'
② Solve y' = slope
③ Find points

①
$y' = \frac{f'g - fg'}{g^2}$

$y' = \frac{1(x-1) - x(1)}{(x-1)^2}$

$= \frac{x-1-x}{(x-1)^2}$

$= \frac{-1}{(x-1)^2}$

② $4y = -x+1$

$y = -\frac{1}{4}x + \frac{1}{4}$

$m = -\frac{1}{4}$

Set $y' = -\frac{1}{4}$

$-\frac{1}{4} = \frac{-1}{(x-1)^2}$

$-(x-1)^2 = -4$

$x-1 = \pm\sqrt{4}$

$x = 1 \pm 2$

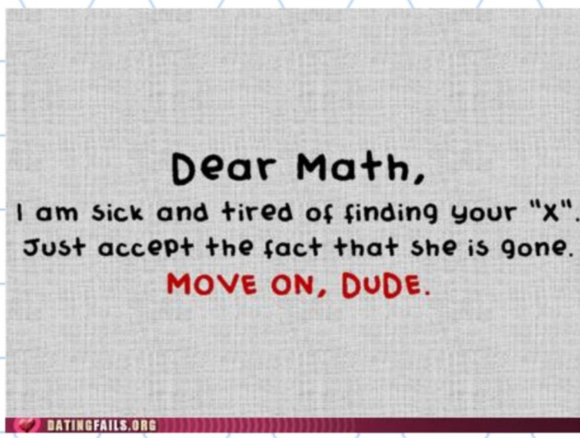
$x = 1+2$
$= 3$

$x = 1-2$
$= -1$

③

$f(3) = \frac{3}{2}$ $\therefore \left(3, \frac{3}{2}\right)$

$f(-1) = \frac{1}{2}$ $\therefore \left(-1, \frac{1}{2}\right)$



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