2.1 The Power Rule

RECAP!!!

What does this limit represent?

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

The slope of the tangent to the graph y=f(x) at the point a.

The instantaneous rate of change of y=f(x) with respect to x at x=a

OR.....

The derivative of f(x) at x=a, written as $f'(a)$

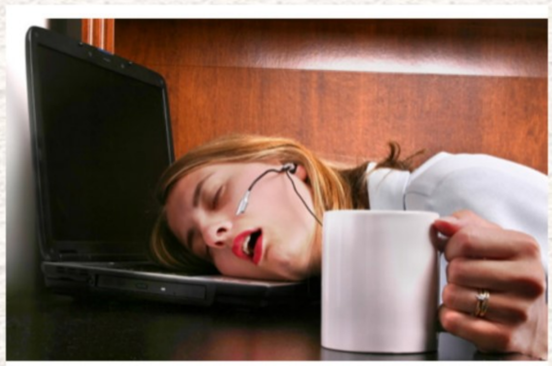
read as "f prime of a"

Recall: The derivative of f(x) with respect to x

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

provided that this limit exists.

The day has arrived when we find an easier way to take the derivative which is not as tedious and time-consuming ...



Proof of the Power Rule

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$, where f(x) = x^n

$$=\lim_{h\to 0}\frac{(x+h)^n-x^n}{h}$$

We will use this formula:
$x^n-a^n=(x-a)(x^{n-1}+x^{n-2}a+\ldots+xa^{n-2}+a^{n-1})$

$$=\lim_{h\to 0}\frac{(x+h-x)\left[(x+h)^{n-1}+(x+h)^{n-2}x+\ldots+(x+h)x^{n-2}+x^{n-1}\right]}{h}$$

$$=\lim_{h\to 0}\frac{(h)\left[(x+h)^{n-1}+(x+h)^{n-2}x+\ldots+(x+h)x^{n-2}+x^{n-1}\right]}{h}$$

$$=\lim_{h\to 0}(x+h)^{n-1}+(x+h)^{n-2}x+\ldots+(x+h)x^{n-2}+x^{n-1}$$

$$=x^{n-1}+x^{n-2}x^1+\ldots+(x)x^{n-2}+x^{n-1}$$

$$=x^{n-1}+x^{n-1}+\ldots+x^{n-1}+x^{n-1}$$

$$=nx^{n-1}$$

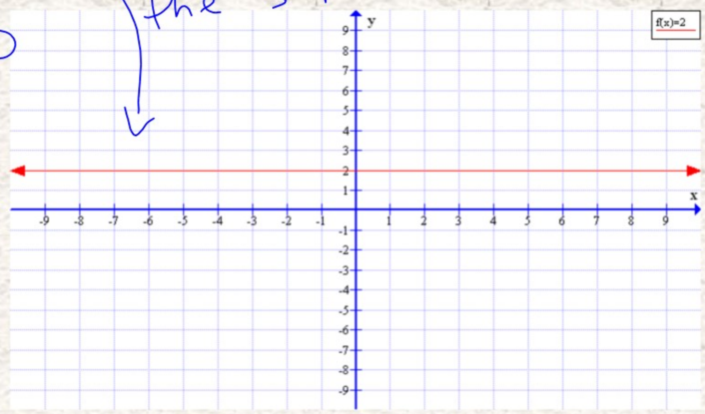
The Power Rule:
If f(x) = x^n, where n is a real number, then f'(x)=nx^{n-1}

What is the derivative of a constant function?

Ex. $f(x)=2$

what's the slope?

$f'(x) = 0$



Does the graph of the derivative change for any constant function? Recall: The derivative is the slope of the tangent at any point.

Ex 1. Find the derivative of the following

a) $f(x) = -7$

$f'(x) = 0$

b) $y= 23\pi$

$y' = 0$

If f is a constant function, $f(x)=c$, then $f'(x)=0$.

In Leibniz notation:

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx} c = 0$

Ex.2 Find the derivative

a) $f(x)= x^4$

$f'(x)=4x^3$

b) $s=t^{-3}$

$s' = -3t^{-4}$

c) $y=3$

$y'=0$

$d) v = t^{\frac{5}{2}}$

$v' = \frac{5}{2}t^{\frac{3}{2}}$

$e) f(x) = \frac{-1}{x^2}$

$= -x^{-2}$

$f'(x) = 2x^{-3}$

Group work

HOW would you solve the following? THINK!

1. Determine the values of x so that the tangent to the function $y = \frac{3}{\sqrt[3]{x}}$ is parallel to the line x + 16y+3=0

- Rewrite: $y = 3x^{-\frac{1}{3}}$
- Find $y' = -x^{-\frac{4}{3}}$
- Set equal to slope $(m = -\frac{1}{16})$
- Solve for x

2. Do the functions $y = \frac{1}{x}$ and $y=x^3$ ever have the same slope?

① Find derivatives
② Set equal and solve

①

$y = x^{-1}$ $\quad\quad$ $y = x^3$

$y' = -x^{-2}$ $\quad\quad$ $y' = 3x^2$

②

$$-x^{-2} = 3x^2$$

$$-\frac{1}{x^2} = 3x^2$$

$$-1 = 3x^4$$

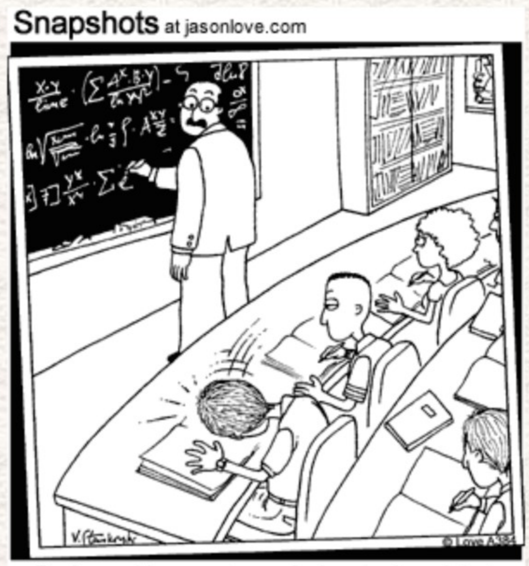
$$\sqrt[4]{-\frac{1}{3}} = x$$

← Imaginary!

∴ Neither slopes are never equal.

HMK!!!

Pg. 82 # 2de, 4a, 7bd, 8c, 9c, 11, 12, 13, 19, 26



Professor Herman stopped when he heard that unmistakable thud -- another brain had imploded.