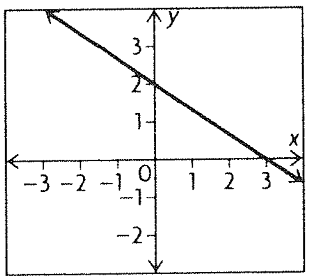
CHAPTER 3
Derivatives and Their Applications

Review of Prerequisite Skills, pp. 116–117

1. a.



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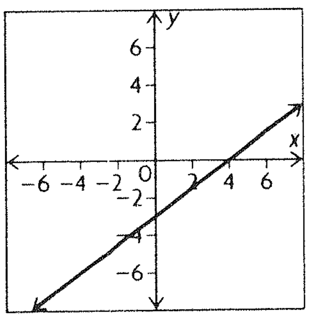

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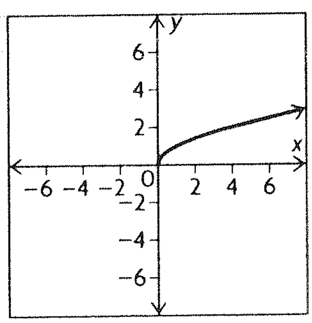

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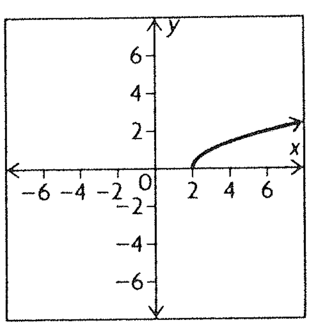

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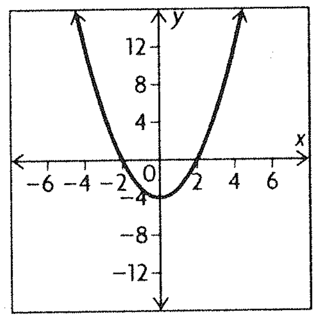

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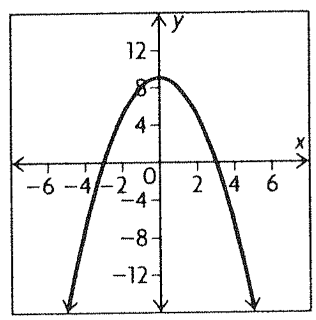


2. a. $3(x-2)+2(x-1)-6=0$

$$3x-6+2x-2-6=0$$
$$5x=14$$
$$x=\frac{14}{5}$$

b. $\frac{1}{3}(x-2)+\frac{2}{5}(x+3)=\frac{x-5}{2}$

$$10(x-2)+12(x+3)=15(x-5)$$
$$10x-20+12x+36=15x-75$$
$$22x+16=15x-75$$
$$7x=-91$$
$$x=-13$$

c. $t^2-4t+3=0$

$$(t-3)(t-1)=0$$
$$t=3 \text{ or } t=1$$

d. $2t^2-5t-3=0$

$$(2t+1)(t-3)=0$$
$$t=-\frac{1}{2} \text{ or } t=3$$

e. $\frac{6}{t}+\frac{t}{2}=4$

$$12+t^2=8t$$
$$t^2-8t+12=0$$
$$(t-6)(t-2)=0$$
$$\therefore t=2 \text{ or } t=6$$

f. $x^3 + 2x^2 - 3x = 0$
$x(x^2 + 2x - 3) = 0$
$x(x + 3)(x - 1) = 0$
$x = 0$ or $x = -3$ or $x = 1$

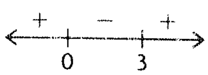
g. $x^3 - 8x^2 + 16x = 0$
$x(x^2 - 8x + 16) = 0$
$x(x - 4)^2 = 0$
$x = 0$ or $x = 4$

h. $4t^3 + 12t^2 - t - 3 = 0$
$4t^2(t + 3) - 1(t + 3) = 0$
$(t + 3)(4t^2 - 1) = 0$
$(t + 3)(2t - 1)(2t + 1) = 0$
$t = -3$ or $t = \frac{1}{2}$ or $t = -\frac{1}{2}$

i. $4t^4 - 13t^2 + 9 = 0$
$(4t^2 - 9)(t^2 - 1) = 0$
$t = \pm\frac{9}{4}$ or $t = \pm 1$

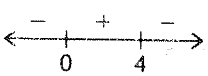
3. a. $3x - 2 > 7$
$3x > 9$
$x > 3$

b. $x(x - 3) > 0$



$x < 0$ or $x > 3$

c. $-x^2 + 4x > 0$



$x(x - 4) < 0$
$0 < x < 4$

4. a. $P = 4s$
$20 = 4s$
$5 = s$
$A = s^2$
$= 5^2$
$= 25 \text{ cm}^2$

b. $A = lw$
$= 8(6) = 48 \text{ cm}^2$

c. $A = \pi r^2$
$= \pi(7)^2$
$= 49\pi \text{ cm}^2$

d. $C = 2\pi r$
$12\pi = 2\pi r$
$6 = r$
$A = \pi r^2$
$= \pi(6)^2$
$= 36\pi \text{ cm}^2$

5. a. $SA = 2\pi rh + 2\pi r^2$
$= 2\pi(4)(3) + 2\pi(4)^2$
$= 24\pi + 32\pi$
$= 56\pi \text{ cm}^2$
$V = \pi r^2 h$
$= \pi(4)^2(3)$
$= 48\pi \text{ cm}^3$

b. $V = \pi r^2 h$
$96\pi = \pi(4)^2 h$
$h = 6 \text{ cm}$
$SA = 2\pi rh + 2\pi r^2$
$= 2\pi(4)(6) + 2\pi(4)^2$
$= 48\pi + 32\pi$
$= 80\pi \text{ cm}^2$

c. $V = \pi r^2 h$
$216\pi = \pi r^2(6)$
$r = 6 \text{ cm}$
$SA = 2\pi rh + 2\pi r^2$
$= 2\pi(6)(6) + 2\pi(6)^2$
$= 72\pi + 72\pi$
$= 144\pi \text{ cm}^2$

d. $SA = 2\pi rh + 2\pi r^2$
$120\pi = 2\pi(5)h + 2\pi(5)^2$
$120\pi = 10\pi h + 50\pi$
$70\pi = 10\pi h$
$h = 7 \text{ cm}$
$V = \pi r^2 h$
$= \pi(5)^2(7)$
$= 175\pi \text{ cm}^3$

6. For a cube, $SA = 6s^2$ and $V = s^3$, where s is the length of any edge of the cube.

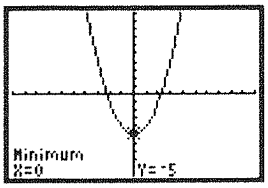
a. $SA = 6(3)^2$
$= 54 \text{ cm}^2$
$V = 3^3$
$= 27 \text{ cm}^3$

b. $SA = 6(\sqrt{5})^2$
$= 30 \text{ cm}^2$
$V = (\sqrt{5})^3$
$= 5\sqrt{5} \text{ cm}^3$

c. $SA = 6(2\sqrt{3})^2$
$= 72 \text{ cm}^2$
$V = (2\sqrt{3})^3$
$= 24\sqrt{3} \text{ cm}^3$

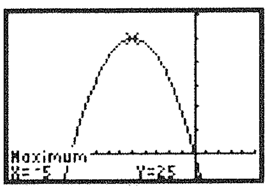
d. $SA = 6(2k)^2$
$= 24k^2 \text{ cm}^2$
$V = (2k)^3$
$= 8k^3 \text{ cm}^3$

7. a. $(3, \infty)$
b. $(-\infty, -2]$
c. $(-\infty, 0)$
d. $[-5, \infty)$
e. $(-2, 8]$
f. $(-4, 4)$
8. a. $\{x \in \mathbf{R} | x > 5\}$
b. $\{x \in \mathbf{R} | x \leq -1\}$
c. $\{x \in \mathbf{R}\}$
d. $\{x \in \mathbf{R} | -10 \leq x \leq 12\}$
e. $\{x \in \mathbf{R} | -1 < x < 3\}$
f. $\{x \in \mathbf{R} | 2 \leq x < 20\}$
9. a.



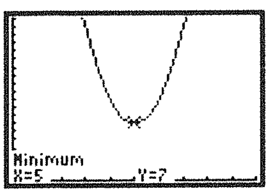
The function has a minimum value of -5 and no maximum value.

b.



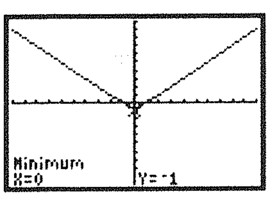
The function has a maximum value of 25 and no minimum value.

c.



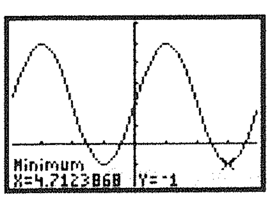
The function has a minimum value of 7 and no maximum value.

d.

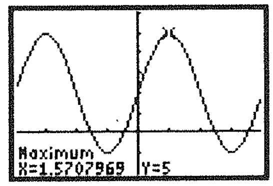


The function has a minimum value of -1 and no maximum value.

e.

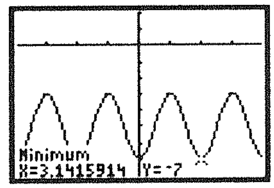


The function has a minimum value of -1.

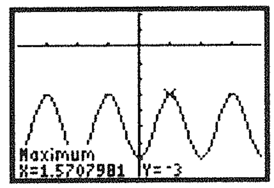


The function has a maximum value of 5.

f.



The function has a minimum value of -7.



The function has a maximum value of -3.

3.1 Higher-Order Derivatives, Velocity, and Acceleration, pp. 127–129

1. $v(1) = 2 - 1 = 1$
$v(5) = 10 - 25 = -15$
At $t = 1$, the velocity is positive; this means that the object is moving in whatever is the positive direction for the scenario. At $t = 5$, the velocity is negative; this means that the object is moving in whatever is the negative direction for the scenario.

2. a. $y = x^{10} + 3x^6$
$y' = 10x^9 + 18x^5$
$y'' = 90x^8 + 90x^4$

b. $f(x) = \sqrt{x} = x^{\frac{1}{2}}$
$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$
$$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$$

c. $y = (1 - x)^2$
$y' = 2(1 - x)(-1)$
$= -2 + 2x$
$y'' = 2$

d. $h(x) = 3x^4 - 4x^3 - 3x^2 - 5$
$h'(x) = 12x^3 - 12x^2 - 6x$
$h''(x) = 36x^2 - 24x - 6$

e. $y = 4x^{\frac{3}{2}} - x^{-2}$
$y' = 6x^{\frac{1}{2}} + 2x^{-3}$

$y'' = 3x^{-\frac{1}{2}} - 6x^{-4}$

$= \frac{3}{\sqrt{x}} - \frac{6}{x^4}$

f. $f(x) = \frac{2x}{x+1}$

$f'(x) = \frac{(x+1)(2) - (2x)(1)}{(x+1)^2}$

$= \frac{2x + 2 - 2x}{(x+1)^2}$

$= \frac{2}{(x+1)^2}$

$f''(x) = \frac{(x+1)^2(0) - (2)(2(x+1))}{(x+1)^4}$

$= \frac{-4x - 4}{(x+1)^4}$

g. $y = x^2 + x^{-2}$

$y' = 2x - 2x^{-3}$

$y'' = 2 + 6x^{-4}$

$= 2 + \frac{6}{x^4}$

h. $g(x) = (3x - 6)^{\frac{1}{2}}$

$g'(x) = \frac{3}{2}(3x - 6)^{-\frac{1}{2}}$

$g''(x) = -\frac{9}{4}(3x - 6)^{-\frac{3}{2}}$

$= -\frac{9}{4(3x - 6)^{\frac{3}{2}}}$

i. $y = (2x + 4)^3$

$y' = 6(2x + 4)^2$

$y'' = 24(2x + 4)$

$= 48x + 96$

j. $h(x) = x^{\frac{5}{3}}$

$h'(x) = \frac{5}{3}x^{\frac{2}{3}}$

$h''(x) = \frac{10}{9}x^{-\frac{1}{3}}$

$= \frac{10}{9x^{\frac{1}{3}}}$

3. a. $s(t) = 5t^2 - 3t + 15$

$v(t) = 10t - 3$

$a(t) = 10$

b. $s(t) = 2t^3 + 36t - 10$

$v(t) = 6t^2 + 36$

$a(t) = 12t$

c. $s(t) = t - 8 + \frac{6}{t}$

$= t - 8 + 6t^{-1}$

$v(t) = 1 - 6t^{-2}$

$a(t) = 12t^{-3}$

d. $s(t) = (t - 3)^2$

$v(t) = 2(t - 3)$

$a(t) = 2$

e. $s(t) = \sqrt{t + 1}$

$v(t) = \frac{1}{2}(t + 1)^{-\frac{1}{2}}$

$a(t) = -\frac{1}{4}(t + 1)^{-\frac{3}{2}}$

f. $s(t) = \frac{9t}{t + 3}$

$v(t) = \frac{9(t + 3) - 9t}{(t + 3)^2}$

$= \frac{27}{(t + 3)^2}$

$a(t) = -54(t + 3)^{-3}$

4. a. i. $t = 3$

ii. $1 < t < 3$

iii. $3 < t < 5$

b. i. $t = 3, t = 7$

ii. $1 < t < 3, 7 < t < 9$

iii. $3 < t < 7$

5. a. $s = \frac{1}{3}t^3 - 2t^2 + 3t$

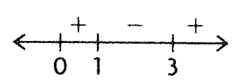
$v = t^2 - 4t + 3$

$a = 2t - 4$

b. For $v = 0$,

$(t - 3)(t - 1) = 0$

$t = 3$ or $t = 1$.



The direction of the motion of the object changes at $t = 1$ and $t = 3$.

c. Initial position is $s(0) = 0$.

Solving,

$0 = \frac{1}{3}t^3 - 2t^2 + 3t$

$= t^3 - 6t^2 + 9t$

$= t(t^2 - 6t + 9)$

$= t(t - 3)^2$

$t = 0$ or $t = 3$

$s = 0$ or $s = 0$.

The object returns to its initial position after 3 s.

6. a. $s = -\frac{1}{3}t^2 + t + 4$

$v = -\frac{2}{3}t + 1$

$v(1) = -\frac{2}{3} + 1$

$= \frac{1}{3}$

$v(4) = -\frac{2}{3}(4) + 1$

$= -\frac{5}{3}$

For $t = 1$, moving in a positive direction.
For $t = 4$, moving in a negative direction.

b. $s(t) = t(t - 3)^2$

$v(t) = (t - 3)^2 + 2t(t - 3)$

$= (t - 3)(t - 3 + 2t)$

$= (t - 3)(3t - 3)$

$= 3(t - 1)(t - 3)$

$v(1) = 0$

$v(4) = 9$

For $t = 1$, the object is stationary.
$t = 4$, the object is moving in a positive direction.

c. $s(t) = t^3 - 7t^2 + 10t$

$v(t) = 3t^2 - 14t + 10$

$v(1) = -1$

$v(4) = 2$

For $t = 1$, the object is moving in a negative direction.
For $t = 4$, the object is moving in a positive direction.

7. a. $s(t) = t^2 - 6t + 8$

$v(t) = 2t - 6$

b. $2t - 6 = 0$

$t = 3\text{ s}$

8. $s(t) = 40t - 5t^2$

$v(t) = 40 - 10t$

a. When $v = 0$, the object stops rising.

$t = 4\text{ s}$

b. Since $s(t)$ represents a quadratic function that opens down because $a = -5 < 0$, a maximum height is attained. It occurs when $v = 0$. Height is a maximum for

$s(4) = 160 - 5(16)$

$= 80\text{ m}.$

9. $s(t) = 8 - 7t + t^2$

$v(t) = -7 + 2t$

$a(t) = 2$

a. $v(5) = -7 + 10$

$= 3\text{ m/s}$

b. $a(5) = 2\text{ m/s}^2$

10. $s(t) = t^{\frac{5}{2}}(7 - t)$

a. $v(t) = \frac{5}{2}t^{\frac{3}{2}}(7 - t) - t^{\frac{5}{2}}$

$= \frac{35}{2}t^{\frac{3}{2}} - \frac{5}{2}t^{\frac{5}{2}} - t^{\frac{5}{2}}$

$= \frac{35}{2}t^{\frac{3}{2}} - \frac{7}{2}t^{\frac{5}{2}}$

$a(t) = \frac{105}{2}t^{\frac{1}{2}} - \frac{35}{4}t^{\frac{3}{2}}$

b. The object stops when its velocity is 0.

$v(t) = \frac{35}{2}t^{\frac{3}{2}} - \frac{7}{2}t^{\frac{5}{2}}$

$= \frac{7}{2}t^{\frac{3}{2}}(5 - t)$

$v(t) = 0$ for $t = 0$ (when it starts moving) and $t = 5$.
So the object stops after 5 s.

c. The direction of the motion changes when its velocity changes from a positive to a negative value or visa versa.

t	$0 \le t < 5$	$t = 5$	$t > 5$
$v(t)$	$(+)(+) = +$	0	$(+)(-) = -$

$v(t) = \frac{7}{2}t^{\frac{3}{2}}(5 - t)$ $\quad v(t) = 0$ for $t = 5$

Therefore, the object changes direction at 5 s.

d. $a(t) = 0$ for $\frac{35}{4}t^{\frac{1}{2}}(6 - t) = 0.$

$t = 0$ or $t = 6$ s.

t	$0 < t < 6$	$t = 6$	$t > 6$
$a(t)$	$(+)(+) = +$	0	$(+)(-) = -$

Therefore, the acceleration is positive for $0 < t < 6$ s.
Note: $t = 0$ yields $a = 0$.

e. At $t = 0$, $s(0) = 0$. Therefore, the object's original position is at 0, the origin.
When $s(t) = 0$,

$t^{\frac{5}{2}}(7 - t) = 0$

$t = 0$ or $t = 7$.

Therefore, the object is back to its original position after 7 s.

11. a. $h(t) = -5t^2 + 25t$

$v(t) = -10t + 25$

$v(0) = 25\text{ m/s}$

b. The maximum height occurs when $v(t) = 0$.

$$-10t + 25 = 0$$
$$t = 2.5 \text{ s}$$
$$h(2.5) = -5(2.5)^2 + 25(2.5)$$
$$= 31.25 \text{ m}$$

c. The ball strikes the ground when $h(t) = 0$.

$$-5t^2 + 25t = 0$$
$$-5t(t - 5) = 0$$
$$t = 0 \text{ or } t = 5$$

The ball strikes the ground at $t = 5$ s.

$$v(5) = -50 + 25$$
$$= -25 \text{ m/s}$$

12. $s(t) = 6t^2 + 2t$

$v(t) = 12t + 2$

$a(t) = 12$

a. $v(8) = 96 + 2 = 98$ m/s

Thus, as the dragster crosses the finish line at $t = 8$ s, the velocity is 98 m/s. Its acceleration is constant throughout the run and equals 12 m/s^2.

b. $s = 60$

$$6t^2 + 2t - 60 = 0$$
$$2(3t^2 + t - 30) = 0$$
$$2(3t + 10)(t - 3) = 0$$
$$t = \frac{-10}{3} \quad \text{or} \quad t = 3$$

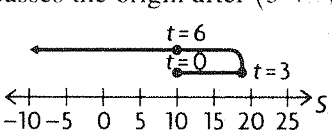
inadmissible $0 \le t \le 8$ $v(3) = 36 + 2 = 38$

Therefore, the dragster was moving at 38 m/s when it was 60 m down the strip.

13. a. $s = 10 + 6t - t^2$

$$v = 6 - 2t$$
$$= 2(3 - t)$$
$$a = -2$$

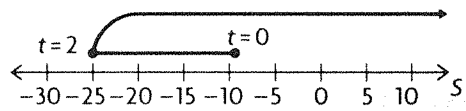
The object moves to the right from its initial position of 10 m from the origin, 0, to the 19 m mark, slowing down at a rate of 2 m/s^2. It stops at the 19 m mark then moves to the left accelerating at 2 m/s^2 as it goes on its journey into the universe. It passes the origin after $(3 + \sqrt{19})$ s.



b. $s = t^3 - 12t - 9$

$$v = 3t^2 - 12$$
$$= 3(t^2 - 4)$$
$$= 3(t - 2)(t + 2)$$
$$a = 6t$$

The object begins at 9 m to the left of the origin, 0, and slows down to a stop after 2 s when it is 25 m to the left of the origin. Then, the object moves to the right accelerating at faster rates as time increases. It passes the origin just before 4 s (approximately 3.7915) and continues to accelerate as time goes by on its journey into space.



14. $s(t) = t^5 - 10t^2$

$$v(t) = 5t^4 - 20t$$
$$a(t) = 20t^3 - 20$$

For $a(t) = 0$,

$$20t^3 - 20 = 0$$
$$20(t^3 - 1) = 0$$
$$t = 1.$$

Therefore, the acceleration will be zero at 1 s.

$$s(1) = 1 - 10$$
$$= -9$$
$$< 0$$
$$v(1) = 5 - 20$$
$$= -15$$
$$< 0$$

Since the signs of both s and v are the same at $t = 1$, the object is moving away from the origin at that time.

15. a. $s(t) = kt^2 + (6k^2 - 10k)t + 2k$

$$v(t) = 2kt + (6k^2 - 10k)$$
$$a(t) = 2k + 0$$
$$= 2k$$

Since $k \neq 0$ and $k \in \mathbf{R}$, then $a(t) = 2k \neq 0$ and an element of the Real numbers. Therefore, the acceleration is constant.

b. For $v(t) = 0$

$$2kt + 6k^2 - 10k = 0$$
$$2kt = 10k - 6k^2$$
$$t = 5 - 3k$$
$$k \neq 0$$

$$s(5 - 3k)$$
$$= k(5 - 3k)^2 + (6k^2 - 10k)(5 - 3k) + 2k$$
$$= k(25 - 30k + 9k^2) + 30k^2 - 18k^3 - 50k + 30k^2 + 2k$$
$$= 25k - 30k^2 + 9k^3 + 30k^2 - 18k^3 - 50k + 30k^2 + 2k$$
$$= -9k^3 + 30k^2 - 23k$$

Therefore, the velocity is 0 at $t = 5 - 3k$, and its position at that time is $-9k^3 + 30k^2 - 23k$.

16. a. The acceleration is continuous at $t = 0$ if $\lim_{t\to 0} a(t) = a(0)$.

For $t \geq 0$,

$$s(t) = \frac{t^3}{t^2 + 1}$$

$$\text{and } v(t) = \frac{3t^2(t^2+1) - 2t(t^3)}{(t^2+1)^2}$$

$$= \frac{t^4 + 3t^2}{(t^2+1)^2}$$

$$\text{and } a(t) = \frac{(4t^3 + 6t)(t^2+1)^2}{(t^2+1)^4}$$

$$- \frac{2(t^2+1)(2t)(t^4+3t^2)}{(t^2+1)^4}$$

$$= \frac{(4t^3+6t)(t^2+1) - 4t(t^4+3t^2)}{(t^2+1)^3}$$

$$= \frac{4t^5 + 6t^3 + 4t^3 + 6t - 4t^5 - 12t^3}{(t^2+1)^3}$$

$$= \frac{-2t^3 + 6t}{(t^2+1)^3}$$

$$\text{Therefore, } a(t) = \begin{cases} 0, \text{ if } t < 0 \\ \dfrac{-2t^3 + 6t}{(t^2+1)^3}, \text{ if } t \geq 0 \end{cases}$$

$$\text{and } v(t) = \begin{cases} 0, \text{ if } t < 0 \\ \dfrac{t^4 + 3t^2}{(t^2+1)^2}, \text{ if } t \geq 0 \end{cases}$$

$$\lim_{t\to 0^-} a(t) = 0, \ \lim_{t\to 0^+} a(t) = \frac{0}{1} = 0.$$

Thus, $\lim_{t\to 0} a(t) = 0$.

Also, $a(0) = \frac{0}{1} = 0$.

Therefore, $\lim_{t\to 0} a(t) = a(0)$.

Thus, the acceleration is continuous at $t = 0$.

b. $$\lim_{t\to +\infty} v(t) = \lim_{t\to +\infty} \frac{t^4 + 3t^2}{t^4 + 2t^2 + 1}$$

$$= \lim_{t\to +\infty} \frac{1 + \frac{3}{t^2}}{1 + \frac{2}{t^2} + \frac{1}{t^4}}$$

$$= 1$$

$$\lim_{t\to +\infty} a(t) = \lim_{t\to\infty} \frac{\frac{-2}{t^3} + \frac{6}{t^4}}{1 + \frac{3}{t^2} + \frac{3}{t^4} + \frac{1}{t^6}}$$

$$= \frac{0}{1}$$

$$= 0$$

17. $v = \sqrt{b^2 + 2gs}$

$$v = (b^2 + 2gs)^{\frac{1}{2}}$$

$$\frac{dv}{dt} = \frac{1}{2}(b^2 + 2gs)^{-\frac{1}{2}} \cdot \left(0 + 2g\frac{ds}{dt}\right)$$

$$a = \frac{1}{2v} \cdot 2gv$$

$$a = g$$

Since g is a constant, a is a constant, as required.

Note: $\frac{ds}{dt} = v$

$\frac{dv}{dt} = a$

18. $F = m_0 \frac{d}{dt}\left(\frac{v}{\sqrt{1 - (\frac{v}{c})^2}}\right)$

Using the quotient rule,

$$= \frac{m_0 \frac{dv}{dt}(1 - \frac{v^2}{c^2})^{\frac{1}{2}} - \frac{1}{2}(1 - \frac{v^2}{c^2})^{-\frac{1}{2}}\left(-\frac{2v\frac{dv}{dt}}{c^2}\right) \cdot v}{1 - \frac{v^2}{c^2}}$$

Since $\frac{dv}{dt} = a$,

$$= \frac{m_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}\left[a\left(1 - \frac{v^2}{c^2}\right) + \frac{v^2 a}{c^2}\right]}{1 - \frac{v^2}{c^2}}$$

$$= \frac{m_0\left[\frac{ac^2 - av^2}{c^2} + \frac{v^2 a}{c^2}\right]}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

$$= \frac{m_0 ac^2}{c^2\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

$$= \frac{m_0 a}{c^2\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}, \text{ as required.}$$

3.2 Maximum and Minimum on an Interval (Extreme Values), pp. 135–138

1. a. The algorithm can be used; the function is continuous.

b. The algorithm cannot be used; the function is discontinuous at $x = 2$.

c. The algorithm cannot be used; the function is discontinuous at $x = 2$.

d. The algorithm can be used; the function is continuous on the given domain.

2. a. max 8; min −12

b. max 30; min −5

c. max 100; min −100

d. max 30; min −20

3. a. $f(x) = x^2 - 4x + 3, 0 \le x \le 3$

$f'(x) = 2x - 4$

Let $2x - 4 = 0$ for max or min

$x = 2$

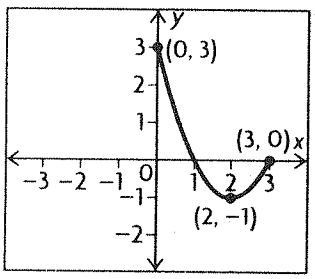
$f(0) = 3$

$f(2) = 4 - 8 + 3 = -1$

$f(3) = 9 - 12 + 3 = 0$

max is 3 at $x = 0$

min is -1 at $x = 2$



b. $f(x) = (x - 2)^2, 0 \le x \le 2$

$f'(x) = 2x - 4$

Let $f'(x) = 0$ for max or min

$2x - 4 = 0$

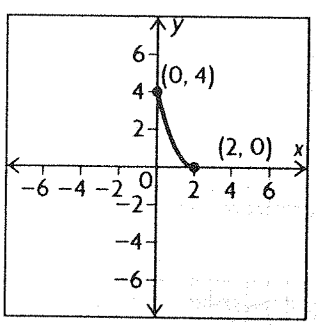
$x = 2$

$f(0) = 4$

$f(2) = 0$

max is 4 at $x = 0$

min is 2 at $x = 2$



c. $f(x) = x^3 - 3x^2, -1 \le x \le 3$

$f'(x) = 3x^2 - 6x$

Let $f'(x) = 0$ for max or min

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ or $x = 2$

$f(-1) = -1 - 3$

$= -4$

$f(0) = 0$

$f(2) = 8 - 12$

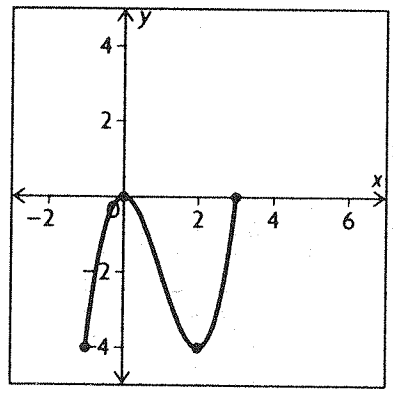
$= -4$

$f(3) = 27 - 27$

$= 0$

min is -4 at $x = -1, 2$

max is 0 at $x = 0, 3$



d. $f(x) = x^3 - 3x^2, x \in [-2, 1]$

$f'(x) = 3x^2 - 6x$

Let $f'(x) = 0$ for max or min

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ or $x = 2$

$x = 2$ is outside the given interval.

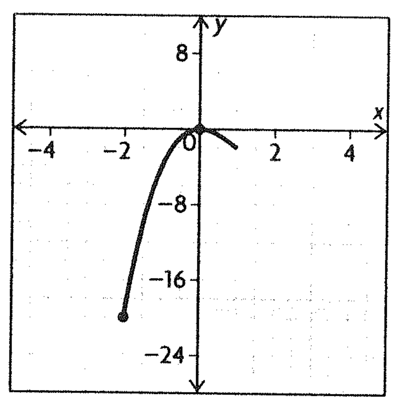
$f(-2) = -20$

$f(0) = 0$

$f(1) = -2$

max is 0 at $x = 0$

min is -20 at $x = -2$



e. $f(x) = 2x^3 - 3x^2 - 12x + 1, x \in [-2, 0]$

$f'(x) = 6x^2 - 6x - 12$

Let $f'(x) = 0$ for max or min

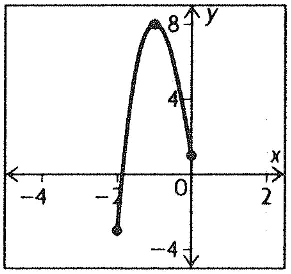
$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

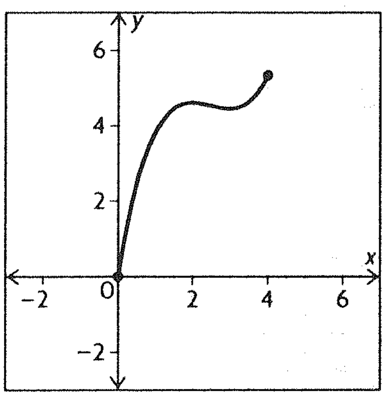
$(x - 2)(x + 1) = 0$

$x = 2$ or $x = -1$

$f(-2) = -16 - 12 + 24 + 1$
$= -3$
$f(-1) = 8$
$f(0) = 1$
$f(2) = \text{not in region}$
max of 8 at $x = -1$
min of -3 at $x = -2$



f. $f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x, x \in [0, 4]$
$f'(x) = x^2 - 5x + 6$
Let $f'(x) = 0$ for max or min
$x^2 - 5x + 6 = 0$
$(x - 2)(x - 3) = 0$
$x = 2$ or $x = 3$
$f(0) = 0$
$f(2) = \frac{14}{3}$
$f(3) = \frac{9}{2}$
$f(4) = \frac{16}{3}$
max is $\frac{16}{3}$ at $x = 4$
min is 0 at $x = 0$



4. a. $f(x) = x + \frac{4}{x}$
$f'(x) = 1 - \frac{4}{x^2}$
$= \frac{x^2 - 4}{x^2}$

Set $f'(x) = 0$ to solve for the critical values.
$\frac{x^2 - 4}{x^2} = 0$
$x^2 - 4 = 0$
$(x - 2)(x + 2) = 0$
$x = 2, x = -2$
Now, evaluate the function, $f(x)$, at the critical values and the endpoints. Note, however, that -2 is not in the domain of the function.
$f(1) = 1 + \frac{4}{1} = 1 + 4 = 5$
$f(2) = 2 + \frac{4}{2} = 2 + 2 = 4$
$f(10) = 10 + \frac{4}{10} = \frac{50}{5} + \frac{2}{5} = \frac{52}{5} = 10.4$
So, the minimum value in the interval is 4 when $x = 2$ and the maximum value is 10.4 when $x = 10$.

b. $f(x) = 4\sqrt{x} - x, 2 \le x \le 9$
$f'(x) = 2x^{-\frac{1}{2}} - 1$
Let $f'(x) = 0$ for max or min
$\frac{2}{\sqrt{x}} - 1 = 0$
$\sqrt{x} = 2$
$x = 4$
$f(2) = 4\sqrt{2} - 2 \doteq 3.6$
$f(4) = 4\sqrt{4} - 4 = 4$
$f(9) = 4\sqrt{9} - 9 = 3$
min value of 3 when $x = 9$
max value of 4 when $x = 4$

c. $f(x) = \frac{1}{x^2 - 2x + 2}, 0 \le x \le 2$
$f'(x) = -(x^2 - 2x + 2)^{-2}(2x - 2)$
$= -\frac{2x - 2}{(x^2 - 2x + 2)^2}$
Let $f'(x) = 0$ for max or min.
$-\frac{2x - 2}{(x^2 - 2x + 2)} = 0$
$2x - 2 = 0$
$x = 1$
$f(0) = \frac{1}{2}, f(1) = 1, f(2) = \frac{1}{2}$
max value of 1 when $x = 1$
min value of $\frac{1}{2}$ when $x = 0, 2$

d. $f(x) = 3x^4 - 4x^3 - 36x^2 + 20$
$f'(x) = 12x^3 - 12x^2 - 72x$
Set $f'(x) = 0$ to solve for the critical values.
$12x^3 - 12x^2 - 72x = 0$
$12x(x^2 - x - 6) = 0$

$12x(x-3)(x+2)=0$

$x=0, x=3, x=-2$

Now, evaluate the function, $f(x)$, at the critical values and the endpoints.

$f(-3)=3(-3)^4-4(-3)^3-36(-3)^2+20=47$

$f(-2)=3(-2)^4-4(-2)^3-36(-2)^2+20=-44$

$f(0)=3(0)^4-4(0)^3-36(0)^2+20=20$

$f(3)=3(3)^4-4(3)^3-36(3)^2+20=-169$

$f(4)=3(4)^4-4(4)^3-36(4)^2+20=-44$

So, the minimum value in the interval is -169 when $x=3$ and the maximum value is 47 when $x=-3$.

e. $f(x)=\frac{4x}{x^2+1}, -2\le x\le 4$

$$f'(x)=\frac{4(x^2+1)-2x(4x)}{(x^2+1)^2}$$

$$=\frac{-4x^2+4}{x^2+1}$$

Let $f'(x)=0$ for max or min:

$-4x^2+4=0$

$x^2=1$

$x=\pm 1$

$f(-2)=\frac{-8}{5}$

$f(-1)=\frac{-4}{2}$

$=-2$

$f(1)=\frac{4}{2}$

$=2$

$f(4)=\frac{16}{17}$

max value of 2 when $x=1$

min value of -2 when $x=-1$

f. Note that part e. is the same function but restricted to a different domain. So, from e. it is seen that the critical points are $x=1$ and $x=-1$.

Now, evaluate the function, $f(x)$, at the critical values and the endpoints. Note, however, that -1 and 1 are not in the domain of the function. Therefore, the only points that need to be checked are the endpoints.

$$f(2)=\frac{4(2)}{(2)^2+1}=\frac{8}{5}=1.6$$

$$f(4)=\frac{4(4)}{(4)^2+1}=\frac{16}{17}\doteq 0.94$$

So, the minimum value in the interval is 0.94 when $x=4$ and the maximum value is 1.6 when $x=2$.

5. a. $v(t)=\frac{4t^2}{4+t^3}, t\ge 0$

Interval $1\le t\le 4$

$v(1)=\frac{4}{5}$

$v(4)=\frac{16}{17}$

$$v'(t)=\frac{(4+t^3)(8t)-4t^2(3t^2)}{(4+t^3)^2}=0$$

$32t+8t^4-12t^4=0$

$-4t(t^3-8)=0$

$t=0, t=2$

$v(2)=\frac{16}{12}=\frac{4}{3}$

max velocity is $\frac{4}{3}$ m/s

min velocity is $\frac{4}{5}$ m/s

b. $v(t)=\frac{4t^2}{1+t^2}$

$$v'(t)=\frac{(1+t^2)(8t)-(4t^2)(2t)}{(1+t^2)^2}$$

$$=\frac{8t+8t^3-8t^3}{(1+t^2)^2}$$

$$=\frac{8t}{(1+t^2)^2}$$

$$\frac{8t}{(1+t^2)^2}=0$$

$8t=0$

$t=0$

$f(0)=0$ is the minimum value that occurs at $x=0$. There is no maximum value on the interval. As x approaches infinity, $f(x)$ approaches the horizontal asymptote $y=4$.

6. $N(t)=30t^2-240t+500$

$N'(t)=60t-240$

$60t-240=0$

$t=4$

$N(0)=500$

$N(4)=30(16)-240(4)+500=20$

$N(7)=30(49)-240(7)+500=290$

The lowest number is 20 bacteria/cm^3.

7. a. $E(v)=\frac{1600v}{v^2+6400}\ 0\le v\le 100$

$$E'(v)=\frac{1600(v^2+6400)-1600v(2v)}{(v^2+6400)^2}$$

Let $E'(N)=0$ for max or min

$1600v^2+6400\times 1600-3200v^2=0$

$1600v^2=6400\times 1600$

$v=\pm 80$

$$E(0) = 0$$
$$E(80) = 10$$
$$E(100) = 9.756$$

The legal speed that maximizes fuel efficiency is 80 km/h.

b. $E(v) = \dfrac{1600v}{v^2 + 6400}\ 0 \le v \le 50$

$$E'(v) = \frac{1600(v^2 + 6400) - 1600v(2v)}{(v^2 + 6400)^2}$$

Let $E'(N) = 0$ for max or min

$$1600v^2 + 6400 \times 1600 - 3200v^2 = 0$$
$$1600v^2 = 6400 \times 1600$$
$$v = \pm 80$$
$$E(0) = 0$$
$$E(50) = 9$$

The legal speed that maximizes fuel efficiency is 50 km/h.

c. The fuel efficiency will be increasing when $E'(v) > 0$. This will show when the slopes of the values of $E(v)$ are positive, and hence increasing. From part a. it is seen that there is one critical value for $v > 0$. This is $v = 80$.

v	slope of $E(v)$
$0 \le v < 80$	$+$
$80 < v \le 100$	$-$

Therefore, within the legal speed limit of 100 km/h, the fuel efficiency E is increasing in the speed interval $0 \le v < 80$.

d. The fuel efficiency will be decreasing when $E'(v) < 0$. This will show when the slopes of the values of $E(v)$ are negative, and hence decreasing. From part a. it is seen that there is one critical value for $v > 0$. This is $v = 80$.

v	slope of $E(v)$
$0 \le v < 80$	$+$
$80 < v \le 100$	$-$

Therefore, within the legal speed limit of 100 km/h, the fuel efficiency E is decreasing in the speed interval $80 < v \le 100$.

8. $C(t) = \dfrac{0.1t}{(t + 3)^2},\ 1 \le t \le 6$

$$C'(t) = \frac{0.1(t + 3)^2 - 0.2t(t + 3)}{(t + 3)^4} = 0$$

$$(t + 3)(0.1t + 0.3 - 0.2t) = 0$$
$$t = 3$$

$C(1) \doteq 0.006\,25$

$C(3) = 0.0083,\ C(6) \doteq 0.0074$

The min concentration is at $t = 1$ and the max concentration is at $t = 3$.

9. $P(t) = 2t + \dfrac{1}{162t + 1},\ 0 \le t \le 1$

$$P'(t) = 2 - (162t + 1)^{-2}(162) = 0$$

$$\frac{162}{(162t + 1)^2} = 2$$

$$81 = 162^2 + t^2 + 324t + 1$$
$$162^2t^2 + 324t - 80 = 0$$
$$81^2t^2 + 81t - 20 = 0$$
$$(81t + 5)(81t - 4) = 0$$
$$t > 0 \quad t = \frac{4}{81}$$
$$= 0.05$$
$$P(0) = 1$$
$$P(0.05) = 0.21$$
$$P(1) = 2.01$$

Pollution is at its lowest level in 0.05 years or approximately 18 days.

10. $r(x) = \dfrac{1}{400}\left(\dfrac{4900}{x} + x\right)$

$$r'(x) = \frac{1}{400}\left(\frac{-4900}{x^2} + 1\right) = 0$$

Let $r'(x) = 0$

$$x^2 = 4900,$$
$$x = 70,\ x > 0$$
$$r(30) = 0.4833$$
$$r(70) = 0.35$$
$$r(120) = 0.402$$

A speed of 70 km/h uses fuel at a rate of 0.35 L/km. Cost of trip is $0.35 \times 200 \times 0.45 = \$31.50$.

11. $f(x) = 0.001x^3 - 0.12x^2 + 3.6x + 10,$
$0 \le x \le 75$

$$f'(x) = 0.003x^2 - 0.24x + 3.6$$

Set $0 = 0.003x^2 - 0.24x + 3.6$

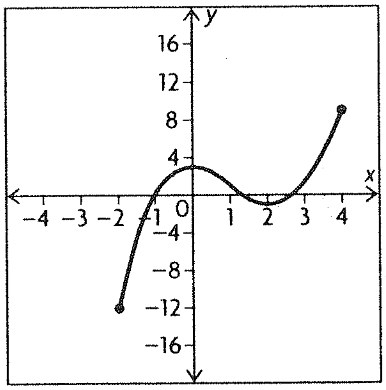
$$x = \frac{0.24 \pm \sqrt{(-0.24)^2 - 4(0.003)(3.6)}}{2(0.003)}$$

$$x = \frac{0.24 \pm 0.12}{0.006}$$

$$x = 60 \text{ or } x = 20$$
$$f(0) = 10$$
$$f(20) = 42$$
$$f(60) = 10$$
$$f(75) = 26.875$$

Absolute max. value $= 42$ at $(20, 42)$ and absolute min. value $= 10$ at $(0, 10)$ and $(60, 10)$.

12. a.



b. D: $-2 \le x \le 4$

c. increasing: $-2 \le x < 0$

$2 < x \le 4$

decreasing: $0 < x < 2$

13. Absolute max.: Compare all local maxima and values of $f(a)$ and $f(b)$ when domain of $f(x)$ is $a \le x \le b$. The one with highest value is the absolute maximum.

Absolute min.: We need to consider all local minima and the value of $f(a)$ and $f(b)$ when the domain of $f(x)$ is $a \le x \le b$. Compare them and the one with the lowest value is the absolute minimum.

You need to check the endpoints because they are not necessarily critical points.

14. $C(x) = 3000 + 9x + 0.05x^2$, $1 \le x \le 300$

$$\text{Unit cost } u(x) = \frac{C(x)}{x}$$
$$= \frac{3000 + 9x + 0.05x^2}{x}$$
$$= \frac{3000}{x} + 9 + 0.05x$$
$$U'(x) = \frac{-3000}{x^2} + 0.05$$

For max or min, let $U'(x) = 0$:

$$0.05x^2 = 3000$$
$$x^2 = 60\,000$$
$$x \doteq 244.9$$
$$U(1) = 3009.05$$
$$U(244) = 33.4950$$
$$U(245) = 33.4948$$
$$U(300) = 34.$$

Production level of 245 units will minimize the unit cost to $33.49.

15. $C(x) = 6000 + 9x + 0.05x^2$

$$U(x) = \frac{C(x)}{x}$$
$$= \frac{6000 + 9x + 0.05x^2}{x}$$
$$= \frac{6000}{x} + 9 + 0.05x$$
$$U'(x) = -\frac{6000}{x^2} + 0.05$$

Set $U'(x) = 0$ and solve for x.

$$-\frac{6000}{x^2} + 0.05 = 0$$
$$0.05 = \frac{6000}{x^2}$$
$$0.05x^2 = 6000$$
$$x^2 = 120\,000$$
$$x \doteq 346.41$$

However, 346.41 is not in the given domain of $1 \le x \le 300$.

Therefore, the only points that need to be checked are the endpoints.

$$f(1) = 6009.05$$
$$f(300) = 44$$

Therefore, a production level of 300 units will minimize the unit cost to $44.

Mid-Chapter Review, pp. 139–140

1. a. $h(x) = 3x^4 - 4x^3 - 3x^2 - 5$

$$h'(x) = 12x^3 - 12x^2 - 6x$$
$$h''(x) = 36x^2 - 24x - 6$$

b. $f(x) = (2x - 5)^3$

$$f'(x) = 6(2x - 5)^2$$
$$f''(x) = 24(2x - 5)$$
$$= 48x - 120$$

c. $y = 15(x + 3)^{-1}$

$$y' = -15(x + 3)^{-2}$$
$$y'' = 30(x + 3)^{-3}$$
$$= \frac{30}{(x + 3)^3}$$

d. $g(x) = (x^2 + 1)^{\frac{1}{2}}$

$$g'(x) = x(x^2 + 1)^{-\frac{1}{2}}$$
$$g''(x) = -x^2(x^2 + 1)^{-\frac{3}{2}} + (x^2 + 1)^{-\frac{1}{2}}$$
$$= -\frac{x^2}{(x^2 + 1)^{\frac{3}{2}}} + \frac{1}{(x^2 + 1)^{\frac{1}{2}}}$$

2. a. $s(3) = (3)^3 - 21(3)^2 + 90(3)$

$$= 27 - 189 + 270$$
$$= 108$$

b. $v(t) = s'(t) = 3t^2 - 42t + 90$

$$v(5) = 3(5)^2 - 42(5) + 90$$

$= 75 - 210 + 90$
$= -45$

c. $a(t) = v'(t) = 6t - 42$
$a(4) = 6(4) - 42$
$= 24 - 42$
$= -18$

3. a. $v(t) = h'(t) = -9.8t + 6$
The initial velocity occurs when time $t = 0$.
$v(0) = -9.8(0) + 6$
$= 6$
So, the initial velocity is 6 m/s.
b. The ball reaches its maximum height when $v(t) = 0$. So set $v(t) = 0$ and solve for t.
$v(t) = 0 = -9.8t + 6$
$9.8t = 6$
$t \doteq 0.61$
Therefore, the ball reaches its maximum height at time $t \doteq 0.61$ s.
c. The ball hits the ground when the height, h, is 0.
$h(t) = 0 = -4.9t^2 + 6t + 2$
$$t = \frac{-6 \pm \sqrt{36 + 39.2}}{-9.8}$$
Taking the negative square root because the value t needs to be positive,
$$t = \frac{-6 - 8.67}{-9.8}$$
$t \doteq 1.50$
So, the ball hits the ground at time $t = 1.50$ s.
d. The question asks for the velocity, $v(t)$, when $t = 1.50$.
$v(1.50) = -9.8(1.50) + 6$
$\doteq -8.67$
Therefore, when the ball hits the ground, the velocity is -8.67 m/s.
e. The acceleration, $a(t)$, is the derivative of the velocity.
$a(t) = v'(t) = -9.8$
This is a constant function. So, the acceleration of the ball at any point in time is -9.8 m/s^2.

4. a. $v(t) = s'(t) = 4 - 14t + 6t^2$
$v(2) = 4 - 14(2) + 6(2)^2$
$= 4 - 28 + 24$
$= 0$
So, the velocity at time $t = 2$ is 0 m/s.
$a(t) = v'(t) = -14 + 12t$
$a(2) = -14 + 12(2)$
$= 10$
So, the acceleration at time $t = 2$ is 10 m/s.
b. The object is stationary when $v(t) = 0$.
$v(t) = 0 = 4 - 14t + 6t^2$
$0 = (6t - 2)(t - 2)$
$t = \frac{1}{3}, t = 2$
Therefore, the object is stationary at time $t = \frac{1}{3}$ s and $t = 2$ s.
Before $t = \frac{1}{3}$, $v(t)$ is positive and therefore the object is moving to the right.
Between $t = \frac{1}{3}$ and $t = 2$, $v(t)$ is negative and therefore the object is moving to the left.
After $t = 2$, $v(t)$ is positive and therefore the object is moving to the right.
c. Set $a(t) = 0$ and solve for t.
$a(t) = 0 = -14 + 12t$
$14 = 12t$
$\frac{7}{6} = t$
$t \doteq 1.2$
So, at time $t \doteq 1.2$ s the acceleration is equal to 0. At that time, the object is neither accelerating nor decelerating.

5. a. $f(x) = x^3 + 3x^2 + 1$
$f'(x) = 3x^2 + 6x$
Set $f'(x) = 0$ to solve for the critical values.
$3x^2 + 6x = 0$
$3x(x + 2) = 0$
$x = 0, x = -2$
Now, evaluate the function, $f(x)$, at the critical values and the endpoints.
$f(-2) = (-2)^3 + 3(-2)^2 + 1 = 5$
$f(0) = (0)^3 + 3(0)^2 + 1 = 1$
$f(2) = (2)^3 + 3(2)^2 + 1 = 21$
So, the minimum value in the interval is 1 when $x = 0$ and the maximum value is 21 when $x = 2$.
b. $f(x) = (x + 2)^2$
$f'(x) = 2(x + 2)$
$= 2x + 4$
Set $f'(x) = 0$ to solve for the critical values.
$2x + 4 = 0$
$2x = -4$
$x = -2$
Now, evaluate the function, $f(x)$, at the critical values and the endpoints.
$f(-3) = (-3 + 2)^2 = (-1)^2 = 1$
$f(-2) = (-2 + 2)^2 = 0$
$f(3) = (3 + 2)^2 = (5)^2 = 25$

So, the minimum value in the interval is 0 when $x = -2$ and the maximum value is 25 when $x = 3$.

c. $f(x) = \frac{1}{x} - \frac{1}{x^3}$

$$f'(x) = -\frac{1}{x^2} + \frac{3}{x^4}$$
$$= \frac{-x^4 + 3x^2}{x^6}$$

Set $f'(x) = 0$ to solve for the critical values.

$$\frac{-x^4 + 3x^2}{x^6} = 0$$
$$-x^4 + 3x^2 = 0$$
$$x^2(-x^2 + 3) = 0$$
$$x = 0$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

Note, however, that $-\sqrt{3}$ and 0 are not in the given domain of the function.

Now, evaluate the function, $f(x)$, at the critical values and the endpoints.

$$f(1) = \frac{1}{1} - \frac{1}{(1)^3} = 1 - 1 = 0$$
$$f(\sqrt{3}) = \frac{1}{\sqrt{3}} - \frac{1}{(\sqrt{3})^3} \doteq 0.38$$
$$f(5) = \frac{1}{5} - \frac{1}{(5)^3} = \frac{24}{125}$$

So, the minimum value in the interval is 0 when $x = 1$ and the maximum value is 0.38 when $x = \sqrt{3}$.

6. The question asks for the maximum temperature of V.

$$V(t) = -0.000\,067t^3 + 0.008\,504\,3t^2 - 0.064\,26t + 999.87$$
$$V'(t) = -0.000\,201t^2 + 0.017\,008\,6t - 0.064\,26$$

Set $V'(t) = 0$ to solve for the critical values.

$$-0.000\,201t^2 + 0.017\,008\,6t - 0.064\,26 = 0$$
$$t^2 - 84.619\,900\,5t + 319.701\,492\,5 = 0$$

Using the quadratic formula,

$t \doteq 3.96$ and $t \doteq 80.66$.

However, 80.66 is not in the domain of the function.

Now, evaluate the function, $V(t)$, at the critical values and the endpoints.

$$V(0) = 999.87$$
$$V(3.96) \doteq 999.74$$
$$V(30) = 1003.79$$

So, the minimum value in the interval is 999.74 when temperature $t = 3.96$.

Therefore, at a temperature of $t = 3.96$°C the volume of water is the greatest in the interval.

7. a. $f(x) = x^4 - 3x$

$$f'(x) = 4x^3 - 3$$
$$f'(3) = 4(3)^3 - 3$$
$$= 105$$

b. $f(x) = 2x^3 + 4x^2 - 5x + 8$

$$f'(x) = 6x^2 + 8x - 5$$
$$f'(-2) = 6(-2)^2 + 8(-2) - 5$$
$$= 3$$

c. $f(x) = -3x^2 - 5x + 7$

$$f'(x) = -6x - 5$$
$$f''(x) = -6$$
$$f''(1) = -6$$

d. $f(x) = 4x^3 - 3x^2 + 2x - 6$

$$f'(x) = 12x^2 - 6x + 2$$
$$f''(x) = 24x - 6$$
$$f''(-3) = 24(-3) - 6$$
$$= -78$$

e. $f(x) = 14x^2 + 3x - 6$

$$f'(x) = 28x + 3$$
$$f'(0) = 28(0) + 3$$
$$= 3$$

f. $f(x) = x^4 + x^5 - x^3$

$$f'(x) = 4x^3 + 5x^4 - 3x^2$$
$$f''(x) = 12x^2 + 20x^3 - 6x$$
$$f''(4) = 12(4)^2 + 20(4)^3 - 6(4)$$
$$= 1448$$

g. $f(x) = -2x^5 + 2x - 6 - 3x^3$

$$f'(x) = -10x^4 + 2 - 9x^2$$
$$f''(x) = -40x^3 - 18x$$
$$f''\left(\frac{1}{3}\right) = -40\left(\frac{1}{3}\right)^3 - 18\left(\frac{1}{3}\right)$$
$$= -\frac{40}{27} - 6$$
$$= -\frac{202}{27}$$

h. $f(x) = -3x^3 - 7x^2 + 4x - 11$

$$f'(x) = -9x^2 - 14x + 4$$
$$f'\left(\frac{3}{4}\right) = -9\left(\frac{3}{4}\right)^2 - 14\left(\frac{3}{4}\right) + 4$$
$$= -\frac{81}{16} - \frac{21}{2} + 4$$
$$= -\frac{185}{16}$$

8. $s(t) = t\left(-\frac{5}{6}t + 1\right)$

$= -\frac{5}{6}t^2 + t$

$s'(t) = -\frac{5}{3}t + 1$

$s''(t) = -\frac{5}{3}$

$\doteq -1.7 \text{ m/s}^2$

9. $s(t) = 189t - t^{\frac{7}{3}}$

a. $s'(t) = 189 - \frac{7}{3}t^{\frac{4}{3}}$

$s'(0) = 189 - \frac{7}{3}(0)^{\frac{4}{3}}$

$= 189 \text{ m/s}$

b. $s'(t) = 0$

$189 - \frac{7}{3}t^{\frac{4}{3}} = 0$

$\frac{7}{3}t^{\frac{4}{3}} = 189$

$t^{\frac{4}{3}} = 81$

$t = \left(81^{\frac{3}{4}}\right)$

$t = 3^3$

$t = 27 \text{ s}$

c. $s(27) = 189(27) - (27)^{\frac{7}{3}}$

$= 5103 - 2187$

$= 2916 \text{ m}$

d. $s''(t) = -\frac{28}{9}t^{\frac{1}{3}}$

$s''(8) = -\frac{28}{9}(8)^{\frac{1}{3}}$

$= -\frac{56}{9}$

$\doteq -6.2 \text{ m/s}^2$

It is decelerating at 6.2 m/s^2.

10. $s(t) = 12t - 4t^{\frac{3}{2}}$

$s'(t) = 12t - 6t^{\frac{1}{2}}$

To find when the stone stops, set $s'(t) = 0$:

$12 - 6t^{\frac{1}{2}} = 0$

$6t^{\frac{1}{2}} = 12$

$t^{\frac{1}{2}} = 2$

$t = (2)^2$

$= 4$

$s(4) = 12(4) - 4(4)^{\frac{3}{2}}$

$= 48 - 32$

$= 16 \text{ m}$

The stone travels 16 m before its stops after 4 s.

11. a. $h(t) = -4.9t^2 + 21t + 0.45$

$0 = -4.9t^2 + 21t + 0.45$

$t = \frac{-21 \pm \sqrt{(21)^2 - 4(-4.9)(0.45)}}{2(-4.9)}$

$t = \frac{-21 \pm \sqrt{449.82}}{-9.8}$

$t \doteq 4.31$ or $t \doteq -0.021$ (rejected since $t \geq 0$)

Note that $h(0) = 0.45 > 0$ because the football is punted from that height. The function is only valid after this point.

Domain: $0 \leq t \leq 4.31$

b. $h(t) = -4.9t^2 + 21t + 0.45$

To determine the domain, find when $h'(t) = 0$.

$h'(t) = -9.8t + 21$

Set $h'(t) = 0$

$0 = -9.8t + 21$

$t \doteq 2.14$

For $0 < t < 2.14$, the height is increasing.

For $2.14 < t < 4.31$, the height is decreasing.

The football will reach its maximum height at 2.14 s.

c. $h(2.14) = -4.9(2.14)^2 + 21(2.14) + 0.45$

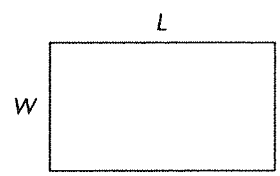
$h(2.14) \doteq -22.44 + 44.94 + 0.45$

$h(2.14) \doteq 22.95$

The football will reach a maximum height of 22.95 m.

3.3 Optimization Problems, pp. 145–147

1.



Let the length be L cm and the width be W cm.

$2(L + W) = 100$

$L + W = 50$

$L = 50 - W$

$A = L \cdot W$

$= (50 - W)(W)$

$A(W) = -W^2 + 50W$ for $0 \leq W \leq 50$

$A'(W) = -2W + 50$

Let $A'(W) = 0$:

$-2W + 50 = 0$

$W = 25$

$A(0) = 0$

$A(25) = 25 \times 25$

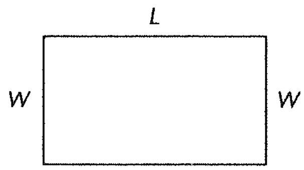
$= 625$

$A(50) = 0.$

The largest area is 625 cm^2 and occurs when $W = 25$ cm and $L = 25$ cm.

2. If the perimeter is fixed, then the figure will be a square.

3.



Let the length of L m and the width W m.

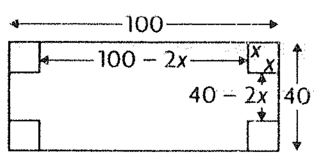
$$2W + L = 600$$
$$L = 600 - 2W$$
$$A = L \cdot W$$
$$= W(600 - 2W)$$
$$A(W) = -2w^2 + 600W, 0 \leq W \leq 300$$
$$A'(W) = -4w + 600$$

For max or min, let $\frac{dA}{dW} = 0$:

$$W = 50$$
$$A(0) = 0$$
$$A(150) = -2(150)^2 + 600 \times 150$$
$$= 45\,000$$
$$A(300) = 0$$

The largest area of 45 000 m^2 occurs when $W = 150$ m and $L = 300$ m.

4. Let dimensions of cut be x cm by x cm. Therefore, the height is x cm.



Length of the box is $100 - 2x$.
Width of the box is $40 - 2x$.
$V = (100 - 2x)(40 - 2x)(x)$ for domain $0 \leq x \leq 20$

Using Algorithm for Extreme Value,

$$\frac{dV}{dx} = (100 - 2x)(40 - 4x) + (40x - 2x^2)(-2)$$
$$= 4000 - 480x + 8x^2 - 80x + 4x^2$$
$$= 12x^2 - 560x + 4000$$

Set $\frac{dV}{dx} = 0$

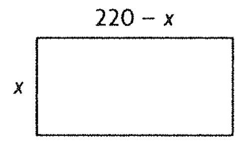
$$3x^2 - 140x + 1000 = 0$$
$$x = \frac{140 \pm \sqrt{7600}}{6}$$
$$x = \frac{140 \pm 128.8}{6}$$
$$x = 8.8 \text{ or } x = 37.9$$

Reject $x = 37.9$ since $0 \leq x \leq 20$

When $x = 0$, $V = 0$
$x = 8.8$, $V = 28\,850$ cm^2
$x = 20$, $V = 0$.

Therefore, the box has a height of 8.8 cm, a length of $100 - 2 \times 8.8 = 82.4$ cm, and a width of $40 - 3 \times 8.8 = 22.4$ cm.

5.



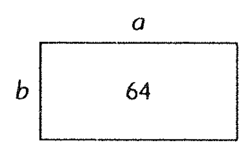
$$A(x) = x(220 - x)$$
$$A(x) = 220x - x^2$$
$$A'(x) = 220 - 2x$$

Set $A'(x) = 0$.

$$0 = 220 - 2x$$
$$x = 110$$
$$220 - 110 = 110$$
$$A'(220) = -220 < 0$$
$$A'(0) = 220 > 0$$

maximum: The dimensions that will maximize the rectangles' area are 110 cm by 110 cm.

6.

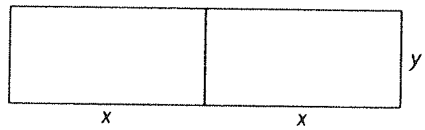


$$ab = 64$$
$$P = 2a + 2b$$
$$P = 2a + 2\left(\frac{64}{a}\right)$$
$$P = 2a + \frac{128}{a}$$
$$P = 2a + 128a^{-1}$$
$$P' = 2 - \frac{128}{a^2}$$

Set $P' = 0$

$$0 = 2 - \frac{128}{a^2}$$
$$2 = \frac{128}{a^2}$$
$$a^2 = 64$$
$$a = 8 \ (-8 \text{ is inadmissible})$$
$$b = \frac{64}{8}$$
$$b = 8$$
$$P'(1) = -126 < 0$$
$$P'(9) \doteq 1.65 > 0$$

maximum: The rectangle should have dimensions 8 m by 8 m.

7. 

Given:

$4x + 3y = 1000$

$$y = \frac{1000 - 4x}{3}$$

$$A = 2xy$$

$$A = 2x\left(\frac{1000 - 4x}{3}\right)$$

$$A = \frac{2000}{3}x - \frac{8}{3}x^2$$

$$A' = \frac{2000}{3} - \frac{16}{3}x$$

Set $A' = 0$

$$0 = \frac{2000}{3} - \frac{16}{3}x$$

$$\frac{16}{3}x = \frac{2000}{3}$$

$$x = 125$$

$$y = \frac{1000 - 4(125)}{3}$$

$$y \doteq 166.67$$

$$A'(250) = -\frac{2000}{3} < 0$$

$$A'(0) = \frac{2000}{3} > 0$$

maximum: The ranger should build the corrals with the dimensions 125 m by 166.67 m to maximize the enclosed area.

8. Netting refers to the area of the rectangular prism. Minimize area while holding the volume constant.

$$V = lwh$$

$$V = x^2y$$

$$144 = x^2y$$

$$y = \frac{144}{x^2}$$

$$A_{\text{Total}} = A_{\text{Side}} + A_{\text{Top}} + A_{\text{Side}} + A_{\text{End}}$$

$$A = xy + xy + xy + x^2$$

$$A = 3xy + x^2$$

$$A = 3x\left(\frac{144}{x^2}\right) + x^2$$

$$A = \frac{432}{x} + x^2$$

$$A = x^2 + 432x^{-1}$$

$$A' = 2x - 432x^{-2}$$

Set $A' = 0$

$$0 = 2x - 432x^{-2}$$

$$2x = 432x^{-2}$$

$$x^3 = 216$$

$$x = 6$$

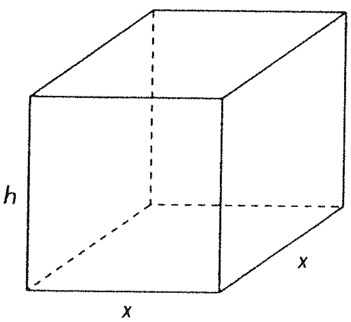
$$y = \frac{144}{6^2}$$

$$y = 4$$

$A'(4) = -19 < 0$

$A'(8) = 9.25 > 0$

minimum: The enclosure should have dimensions 4 m × 6 m × 6 m.

9. 

Let the base be x by x and the height be h

$$x^2h = 1000$$

$$h = \frac{1000}{x^2} \qquad (1)$$

Surface area $= 2x^2 + 4xh$

$$A = 2x^2 + 4xh \qquad (2)$$

$$= 2x^2 + 4x\left(\frac{1000}{x^2}\right)$$

$$= 2x^2 + \frac{4000}{x} \text{ for domain } 0 \le x \le 10\sqrt{2}$$

Using the max min Algorithm,

$$\frac{dA}{dx} = 4x - \frac{4000}{x^2} = 0$$

$x \ne 0$, $4x^3 = 4000$

$$x^3 = 1000$$

$$x = 10$$

$A = 200 + 400 = 600 \text{ cm}^2$

Step 2: At $x \to 0$, $A \to \infty$

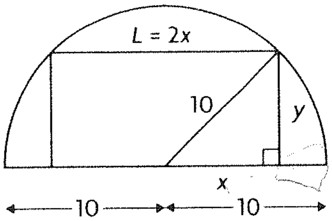
Step 3: At $x = 10\sqrt{10}$,

$$A = 2000 + \frac{4000}{10\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$$

$$= 2000 + 40\sqrt{10}$$

Minimum area is 600 cm^2 when the base of the box is 10 cm by 10 cm and height is 10 cm.

10.



Let the length be $2x$ and the height be y. We know $x^2 + y^2 = 100$.

$$y = \pm\sqrt{100 - x^2}$$

Omit negative area $= 2xy$

$$= 2x\sqrt{100 - x^2}$$

for domain $0 \leq x \leq 10$

Using the max min Algorithm,

$$\frac{dA}{dx} = 2\sqrt{100 - x^2} + 2y \cdot \frac{1}{2}(100 - x^2)^{-\frac{1}{2}}(-2x).$$

Let $\frac{dA}{dx} = 0$.

$$2\sqrt{100 - x^2} - \frac{2x^2}{\sqrt{100 - x^2}} = 0$$

$$2(100 - x^2) - 2x^2 = 0$$

$$100 = 2x^2$$

$$x^2 = 50$$

$x = 5\sqrt{2}, x > 0$. Thus, $y = 5\sqrt{2}$, $L = 10\sqrt{2}$

Part 2: If $x = 0$, $A = 0$

Part 3: If $x = 10$, $A = 0$

The largest area occurs when $x = 5\sqrt{2}$ and the area is $10\sqrt{2}\sqrt{100 - 50}$

$= 10\sqrt{2}\sqrt{50}$

$= 100$ square units.

11. a. Let the radius be r cm and the height be h cm.

Then $\pi r^2 h = 1000$

$$h = \frac{1000}{\pi r^2}$$

Surface Area: $A = 2\pi r^2 + 2\pi rh$

$$= 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$$

$$= 2\pi r^2 + \frac{2000}{4}, 0 \leq r \leq \infty$$

$$\frac{dA}{dr} = 4\pi r - \frac{2000}{r^2}$$

For max or min, let $\frac{dA}{dr} = 0$.

$$4\pi r - \frac{2000}{r^2} = 0$$

$$r^3 = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}} \doteq 5.42$$

When $r = 0$, $A \to \infty$

$r = 5.42$ $A \doteq 660.8$

$r \to \infty$, $A \to \infty$

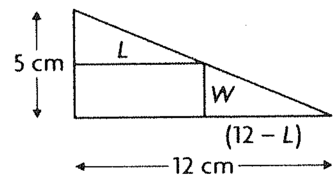
The minimum surface area is approximately $661\ cm^3$ when $r = 5.42$.

b. $r = 5.42$, $h = \frac{1000}{\pi(5.42)^2} \doteq 10.84$

$$\frac{h}{d} = \frac{10.84}{2 \times 5.42} = \frac{1}{1}$$

Yes, the can has dimensions that are larger than the smallest that the market will accept.

12. a.



Let the rectangle have length L cm on the 12 cm leg and width W cm on the 5 cm leg.

$A = LW$

By similar triangles, $\frac{12 - L}{12} = \frac{W}{5}$

$$60 - 5L = 12W$$

$$L = \frac{60 - 12W}{5}$$

$A = \frac{(60 - 12W)W}{5}$ for domain $0 \leq W \leq 5$

Using the max min Algorithm.

$\frac{dA}{dW} = \frac{1}{5}[60 - 24W] = 0$, $W = \frac{60}{24} = 2.5$ cm.

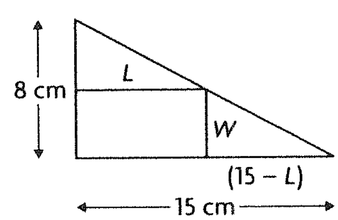
When $W = 2.5$ cm, $A = \frac{(60 - 30) \times 2.5}{5} = 15\ cm^2$.

Step 2: If $W = 0$, $A = 0$

Step 3: If $W = 5$, $A = 0$

The largest possible area is $15\ cm^2$ and occurs when $W = 2.5$ cm and $L = 6$ cm.

b.



Let the rectangle have length L cm on the 15 cm leg and width W cm on the 8 cm leg.

$A = LW$ ①

By similar triangles, $\frac{15 - L}{15} = \frac{W}{8}$

$$120 - 8L = 15W$$

$L = \frac{120 - 15W}{8}$ ②

$A = \frac{(120 - 15W)W}{8}$ for domain $0 \le W \le 8$

Using the max min Algorithm,

$\frac{dA}{dW} = \frac{1}{8}[120 - 30W] = 0, W = \frac{120}{30} = 4$ cm.

When $W = 4$ cm, $A = \frac{(120 - 60) \times 4}{8} = 30\text{ cm}^2$.

Step 2: If $W = 0, A = 0$

Step 3: If $W = 8, A = 0$

The largest possible area is 30 cm^2 and occurs when $W = 4$ cm and $L = 7.5$ cm.

c. The largest area occurs when the length and width are each equal to one-half of the sides adjacent to the right angle.

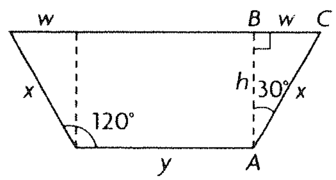
13. a. Let the base be y cm, each side x cm and the height h cm.

$2x + y = 60$

$y = 60 - 2x$

$A = yh + 2 \times \frac{1}{2}(wh)$

$= yh + wh$



From $\triangle ABC$

$\frac{h}{x} = \cos 30°$

$h = x \cos 30°$

$= \frac{\sqrt{3}}{2}x$

$\frac{w}{x} = \sin 30°$

$w = x \sin 30°$

$= \frac{1}{2}x$

Therefore, $A = (60 - 2x)\left(\frac{\sqrt{3}}{2}x\right) + \frac{x}{2} \times \frac{\sqrt{3}}{2}x$

$A(x) = 30\sqrt{3}x - \sqrt{3}x^2 + \frac{\sqrt{3}}{4}x^2, 0 \le x \le 30$

Apply the Algorithm for Extreme Values,

$A'(x) = 30\sqrt{3} - 2\sqrt{3}x + \frac{\sqrt{3}}{2}x$

Now, set $A'(x) = 0$

$30\sqrt{3} - 2\sqrt{3} + \frac{\sqrt{3}}{2}x = 0.$

Divide by $\sqrt{3}$:

$30 - 2x + \frac{x}{2} = 0$

$x = 20.$

To find the largest area, substitute $x = 0, 20,$ and 30.

$A(0) = 0$

$A(20) = 30\sqrt{3}(20) - \sqrt{3}(20)^2 + \frac{\sqrt{3}}{4}(20)^2$

$= 520$

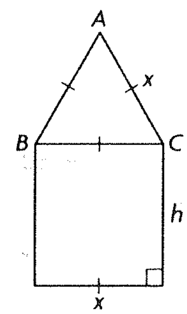
$A(30) = 30\sqrt{3}(30) - \sqrt{3}(30)^2 + \frac{\sqrt{3}}{4}(30)^2$

$\doteq 390$

The maximum area is 520 cm^2 when the base is 20 cm and each side is 20 cm.

b. Multiply the cross-sectional area by the length of the gutter, 500 cm. The maximum volume that can be held by this gutter is approximately 500(520) or 260 000 cm^3.

14. a.



$4x + 2h = 6$

$2x + h = 3$ or $h = 3 - 2x$

$\text{Area} = xh + \frac{1}{2} \times x \times \frac{\sqrt{3}}{2}x$

$= x(3 - 2x) + \frac{\sqrt{3}x^2}{4}$

$A(x) = 3x - 2x^2 + \frac{\sqrt{3}}{4}x^2$

$A'(x) = 3 - 4x + \frac{\sqrt{3}}{2}x, 0 \le x \le 1.5$

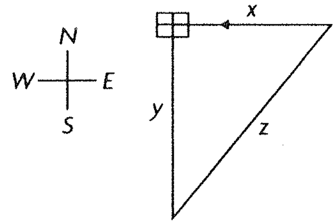
For max or min, let $A'(x) = 0, x \doteq 1.04.$

$A(0) = 0, A(1.04) \doteq 1.43, A(1.5) \doteq 1.42$

The maximum area is approximately 1.43 cm^2 and occurs when $x = 0.96$ cm and $h = 1.09$ cm.

b. Yes. All the wood would be used for the outer frame.

15.



Let z represent the distance between the two trains. After t hours, $y = 60t$, $x = 45(1 - t)$

$$z^2 = 3600t^2 + 45^2(1 - t)^2, 0 \leq t \leq 1$$

$$2z\frac{dz}{dt} = 7200t - 4050(1 - t)$$

$$\frac{dz}{dt} = \frac{7200t - 4050(1 - t)}{2z}$$

For max or min, let $\frac{dz}{dt} = 0$.

$$7200t - 4050(1 - t) = 0$$
$$t = 0.36$$

When $t = 0$, $z^2 = 45^2$, $z = 45$

$t = 0.36$, $z^2 = 3600(0.36)^2 + 45^2(1 - 0.36)^2$

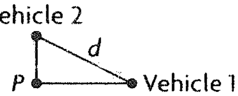
$z^2 = 129$

$z = 36$

$t = 1$, $z^2 = \sqrt{3600} = 60$

The closest distance between the trains is 36 km and occurs at 0.36 h after the first train left the station.

16.



At any time after 1:00 p.m., the distance between the first vehicle and the second vehicle is the hypotenuse of a right triangle, where one side of the triangle is the distance from the first vehicle to P and the other side is the distance from the second vehicle to P. The distance between them is therefore $d = \sqrt{(60t)^2 + (5 - 80t)^2}$ where t is the time in hours after 1:00. To find the time when they are closest together, d must be minimized.

$$d = \sqrt{(60t)^2 + (5 - 80t)^2}$$

$$d = \sqrt{3600t^2 + 25 - 800t + 6400t^2}$$

$$d = \sqrt{10\,000t^2 + 25 - 800t}$$

$$d' = \frac{20\,000t - 800}{2\sqrt{10\,000t^2 + 25 - 800t}}$$

Let $d' = 0$:

$$\frac{20\,000t - 800}{2\sqrt{10\,000t^2 + 25 - 800t}} = 0$$

Therefore $20\,000t - 800 = 0$

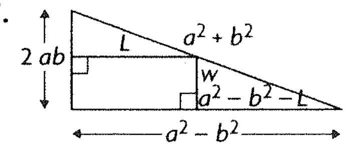
$$20\,000t = 800$$
$$t = .04 \text{ hours}$$

There is a critical number at $t = .04$ hours

v	$t < .04$	.04	$t > .04$
$d'(t)$	−	0	+
Graph	Dec.	Local Min	Inc.

There is a local minimum at $t = .04$, so the two vehicles are closest together .04 hours after 1:00, or 1:02. The distance between them at that time is 3 km.

17.



$$\frac{a^2 - b^2 - L}{a^2 - b^2} = \frac{W}{2ab}$$

$$W = \frac{2ab}{a^2 - b^2}(a^2 - b^2 - L)$$

$$A = LW = \frac{2ab}{a^2 - b^2}[a^2L - b^2L - L^2]$$

Let $\frac{dA}{dL} = a^2 - b^2 - 2L = 0$.

$$L = \frac{a^2 - b^2}{2}$$

and $W = \frac{2ab}{a^2 - b^2}\left[a^2 - b^2 - \frac{a^2 - b^2}{2}\right]$

$= ab$.

The hypothesis is proven.

18. Let the height be h and the radius r.

Then, $\pi r^2 h = k$, $h = \frac{k}{\pi r^2}$.

Let M represent the amount of material.

$$M = 2\pi r^2 + 2\pi rh$$
$$= 2\pi r^2 + 2\pi rh\left(\frac{k}{\pi r^2}\right)$$
$$= 2\pi r^2 + \frac{2k}{r}, 0 \leq r \leq \infty$$

Using the max min Algorithm,

$$\frac{dM}{dr} = 4\pi r - \frac{2k}{r^2}$$

Let $\frac{dM}{dr} = 0$, $r^3 = \frac{k}{2\pi}$, $r \neq 0$ or $r = \left(\frac{k}{2\pi}\right)^{\frac{1}{3}}$.

When $r \to 0$, $M \to \infty$

$r \to \infty$, $M \to \infty$

$$r = \left(\frac{k}{2\pi}\right)^{\frac{1}{3}}$$

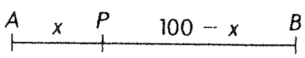
$$d = 2\left(\frac{k}{2\pi}\right)^{\frac{1}{3}}$$

$$h = \frac{k}{\pi\left(\frac{k}{2\pi}\right)^{\frac{2}{3}}} = \frac{k}{\pi} \cdot \frac{(2\pi)^{\frac{2}{3}}}{\ } = \frac{k^{\frac{1}{3}}}{\ } \cdot 2^{\frac{2}{3}}$$

Min amount of material is

$$M = 2\pi\left(\frac{k}{2\pi}\right)^{\frac{2}{3}} + 2k\left(\frac{2\pi}{k}\right)^{\frac{1}{3}}.$$

$$\text{Ratio } \frac{h}{d} = \frac{\left(\frac{k}{\pi}\right)^{\frac{1}{3}} \cdot 2^{\frac{2}{3}}}{2\left(\frac{k}{2\pi}\right)^{\frac{1}{3}}} = \frac{\left(\frac{k}{\pi}\right)^{\frac{1}{3}} \cdot 2^{\frac{2}{3}}}{2^{\frac{2}{3}}\left(\frac{k}{\pi}\right)^{\frac{1}{3}}} = \frac{1}{1}$$

19. 

Cut the wire at P and label diagram as shown. Let AP form the circle and PB the square.

Then, $2\pi r = x$

$$r = \frac{x}{2\pi}$$

And the length of each side of the square is $\frac{100 - x}{4}$.

$$\text{Area of circle} = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$$

$$\text{Area of square} = \left(\frac{100 - x}{4}\right)^2$$

The total area is

$$A(x) = \frac{x^2}{4\pi} + \left(\frac{100 - x}{4}\right)^2, \text{ where } 0 \le x \le 100.$$

$$A'(x) = \frac{2x}{4\pi} + 2\left(\frac{100 - x}{4}\right)\left(-\frac{1}{4}\right) = \frac{x}{2\pi} - \frac{100 - x}{8}$$

For max or min, let $A'(x) = 0$.

$$\frac{x}{2\pi} - \frac{100 - x}{8} = 0$$

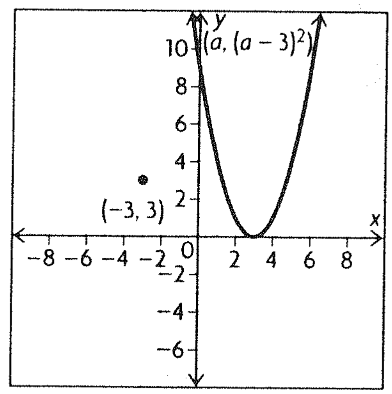
$$x = \frac{100\pi}{r} + \pi \doteq 44$$

$$A(0) = 625$$

$$A(44) = \frac{44^2}{4\pi} + \left(\frac{100 - 44}{4}\right)^2 \doteq 350$$

$$A(100) = \frac{100^2}{4\pi} \doteq 796$$

a. The maximum area is 796 cm^2 and occurs when all of the wire is used to form a circle.
b. The minimum area is 350 cm^2 when a piece of wire of approximately 44 cm is bent into a circle.

20. 

Any point on the curve can be represented by $(a, (a - 3)^2)$.
The distance from $(-3, 3)$ to a point on the curve is $d = \sqrt{(a + 3)^2 + ((a - 3)^2 - 3)^2}$.
To minimize the distance, we consider the function $d(a) = (a + 3)^2 + (a^2 - 6a + 6)^2$.
in minimizing $d(a)$, we minimize d since $d > 1$ always.
For critical points, set $d'(a) = 0$.
$d'(a) = 2(a + 3) + 2(a^2 - 6a + 6)(2a - 6)$
if $d'(a) = 0$,

$$a + 3 + (a^2 - 6a + 6)(2a - 6) = 0$$
$$2a^3 - 18a^2 + 49a - 33 = 0$$
$$(a - 1)(2a^2 - 16a + 33) = 0$$

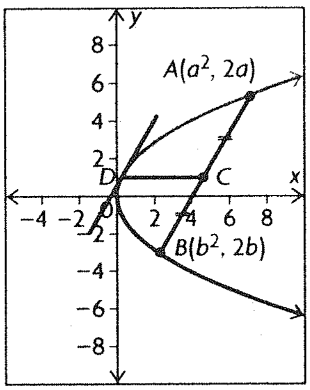
$$a = 1, \text{ or } a = \frac{16 \pm \sqrt{-8}}{4}$$

There is only one critical value, $a = 1$.
To determine whether $a = 1$ gives a minimal value, we use the second derivative test:

$d'(a) = 6a^2 - 36a + 49$
$d''(1) = 6 - 36 + 49$
≥ 0.
Then, $d(1) = 4^2 + 1^2$
$= 17$.

The minimal distance is $d = \sqrt{17}$, and the point on the curve giving this result is $(1, 4)$.

21.



Let the point A have coordinates $(a^2, 2a)$. (Note that the x-coordinate of any point on the curve is positive, but that the y-coordinate can be positive or negative. By letting the x-coordinate be a^2, we eliminate this concern.) Similarly, let B have coordinates $(b^2, 2b)$.

The slope of AB is

$$\frac{2a - 2b}{a^2 - b^2} = \frac{2}{a + b}.$$

Using the mid-point property, C has coordinates

$$\left(\frac{a^2 + b^2}{2}, a + b\right).$$

Since CD is parallel to the x-axis, the y-coordinate of D is also $a + b$. The slope of the tangent at D is given by $\frac{dy}{dx}$ for the expression $y^2 = 4x$.

Differentiating,

$$2y\frac{dy}{dx} = 4$$

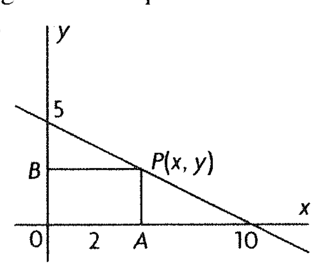
$$\frac{dy}{dx} = \frac{2}{y}$$

And since at point D, $y = a + b$,

$$\frac{dy}{dx} = \frac{2}{a + b}.$$

But this is the same as the slope of AB. Then, the tangent at D is parallel to the chord AB.

22.



Let the point $P(x, y)$ be on the line $x + 2y - 10 = 0$.

Area of $\triangle APB = xy$

$x + 2y = 10$ or $x = 10 - 2y$

$A(y) = (10 - 2y)y$

$\quad = 10y - 2y^2, 0 \le y \le 5$

$A'(y) = 10.4y$

For max or min, let $A'(y) = 0$ or $10 - 4y = 0$, $y = 2.5$,

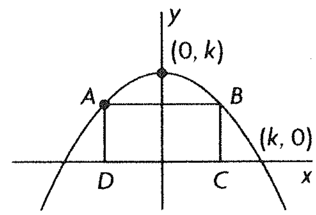
$A(0) = 0$

$A(2.5) = (10 - 5)(2.5) = 12.5$

$A(5) = 0.$

The largest area is 12.5 units squared and occurs when P is at the point $(5, 2.5)$.

23.



A is $(-x, y)$ and $B(x, y)$

Area $= 2xy$ where $y = k^2 - x^2$

$A(x) = 2x(k^2 - x^2)$

$\quad = 2k^2x - 2x^3, -k \le x \le k$

$A'(x) = 2k^2 - 6x^2$

For max or min, let $A'(x) = 0$,

$6x^2 = 2k^2$

$$x = \pm\frac{k}{\sqrt{3}}$$

When $x = \pm\frac{k}{\sqrt{3}}$, $y = k^2 - \left(\frac{k}{\sqrt{3}}\right)^2 = \frac{2}{3}k^2$

Max area is $A = \frac{2k}{\sqrt{3}} \times \frac{2}{3}k^2 = \frac{4k^3}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{4k^3}{9}$ square units.

3.4 Optimization Problems in Economics and Science, pp. 151–154

1. a. $C(625) = 75(\sqrt{625} - 10)$

$\quad = 1125$

Average cost is $\frac{1125}{625} = \$1.80$.

b. $C(x) = 75(\sqrt{x} - 10)$

$\quad = 75\sqrt{x} - 750$

$$C'(x) = \frac{75}{2\sqrt{x}}$$

$$C'(1225) = \frac{75}{2\sqrt{1225}} = \$1.07$$

c. For a marginal cost of $\$0.50/L$,

$$\frac{75}{2\sqrt{x}} = 0.5$$

$75 = \sqrt{x}$

$x = 5625$

The amount of product is 5625 L.

2. $N(t) = 20t - t^2$

a. $N(3) = 60 - 9$
$= 51$
$N(2) = 40 - 4$
$= 36$
$51 - 36 = 15$ terms

b. $N'(t) = 20 - 2t$
$N'(2) = 20 - 4$
$= 16$ terms/h

c. $t > 0$, so the maximum rate (maximum value of $N'(t)$) is 20. 20 terms/h

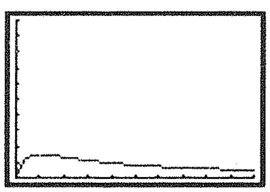
3. $L(t) = \dfrac{6t}{t^2 + 2t + 1}$

a. $L'(t) = \dfrac{6(t^2 + 2t + 1) - 6t(2t + 2)}{(t^2 + 2t + 1)^2}$
$= \dfrac{-6t^2 + 6}{(t^2 + 2t + 1)^2}$

Let $L'(t) = 0$, then $-6t^2 + 6 = 0$,
$t^2 = 1$
$t^2 = \pm 1$.

b. $L(1) = \dfrac{6}{1 + 2 + 1} = \dfrac{6}{4} = 1.5$

c.



d. The level will be a maximum.

e. The level is decreasing.

4. $C = 4000 + \dfrac{h}{15} + \dfrac{15\,000\,000}{h}$, $1000 \le h \le 20\,000$

$\dfrac{dC}{dh} = \dfrac{1}{15} - \dfrac{15\,000\,000}{h^2}$

Set $\dfrac{dC}{dh} = 0$, therefore, $\dfrac{1}{15} - \dfrac{15\,000\,000}{h^2} = 0$,
$h^2 = 225\,000\,000$
$h = 15\,000, h > 0$.

Using the max min Algorithm, $1000 \le h \le 20\,000$.

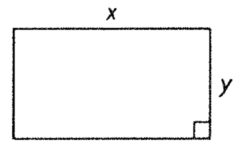
When $h = 1000$, $C = 4000 + \dfrac{1000}{15} + \dfrac{15\,000\,000}{1000}$,
$\doteq 19\,067$.

When $h = 15\,000$, $C = 4000 + \dfrac{15\,000}{15} + \dfrac{15\,000\,000}{15\,000}$
$= 6000$.

When $h = 20\,000$, $C \doteq 6083$.

The minimum operating cost of \$6000/h occurs when the plane is flying at 15 000 m.

5.



Label diagram as shown and let the side of length x cost \$6/m and the side of length y be \$9/m.

Therefore, $(2x)(6) + (2y)(9) = 9000$
$2x + 3y = 1500$.

Area $A = xy$

But $y = \dfrac{1500 - 2x}{3}$.

$A(x) = x\left(\dfrac{1500 - 2x}{3}\right)$
$= 500x - \dfrac{2}{3}x^2$ for domain $0 \le x \le 500$

$A'(x) = 500 - \dfrac{4}{3}x$

Let $A'(x) = 0$, $x = 375$.

Using max min Algorithm, $0 \le x \le 500$,

$A(0) = 0$, $A(375) = 500(375) - \dfrac{2}{3}(375)^2$
$= 93\,750$
$A(500) = 0$.

The largest area is 93 750 m^2 when the width is 250 m by 375 m.

6. Let x be the number of \$25 increases in rent.

$P(x) = (900 + 25x)(50 - x) - (50 - x)(75)$
$P(x) = (50 - x)(825 + 25x)$
$P(x) = 41\,250 + 1250x - 825x - 25x^2$
$P(x) = 41\,250 + 425x - 25x^2$
$P'(x) = 425 - 50x$

Set $P'(x) = 0$
$0 = 425 - 50x$
$50x = 425$
$x = 8.5$
$x = 8$ or $x = 9$
$P'(0) = 425 > 0$
$P'(10) = -75 < 0$

maximum: The real estate office should charge \$900 + \$25(8) = \$1100 or \$900 + \$25(9) = \$1125 rent to maximize profits. Both prices yield the same profit margin.

7. Let the number of fare changes be x. Now, ticket price is \$20 + \$0.5x. The number of passengers is $10\,000 - 200x$.

The revenue $R(x) = (10\,000 - 200x)(20 + 0.5x)$,
$R(x) = -200(20 + 0.5x) + 0.5(1000 - 200x)$
$= -4000 - 100x + 5000 - 100x$.

Let $R'(x) = 0$:

$200x = 1000$

$x = 5.$

The new fare is \$20 + \$0.5(5) = \$22.50 and the maximum revenue is \$202 500.

8. Cost $C - \left(\frac{v^3}{2} + 216\right) \times t$

Where $vt = 500$ or $t = \frac{500}{v}$.

$$C(v) = \left(\frac{v^3}{2} + 216\right)\left(\frac{500}{v}\right)$$

$$= 250v^2 + \frac{108\,000}{v}, \text{ where } v \geq 0.$$

$$C'(v) = 500v - \frac{108\,000}{v^2}$$

Let $C'(v) = 0$, then $500v = \frac{108\,000}{v^2}$

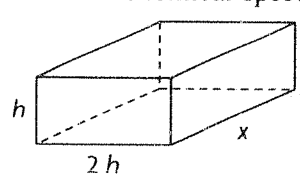
$$v^3 = \frac{108\,000}{500}$$

$$v^3 = 216$$

$$v = 6.$$

The most economical speed is 6 nautical miles/h.

9.



Label diagram as shown.

We know that $(x)(2h)(h) = 20\,000$

or $h^2x = 10\,000$

$$x = \frac{10\,000}{h^2}$$

Cost $C = 40(2hx) + 2xh(200)$

$+ 100(2)(2h^2 + xh)$

$= 80xh + 400xh + 400h^2 + 200xh$

$= 680xh + 400h^2$

Since $x = \frac{10\,000}{h^2}$,

$$C(h) = 680h\left(\frac{10\,000}{h^2}\right) + 400h^2, 0 \leq h \leq 100$$

$$C(h) = \frac{6\,800\,000}{h} + 400h^2$$

$$C'(h) = \frac{6\,800\,000}{h^2} + 800h.$$

Let $C'(h) = 0$,

$800h^3 = 6\,800\,000$

$h^3 = 8500$

$h \doteq 20.4.$

Apply max min Algorithm.

as $h \to 0$ $C(0) \to \infty$

$$C(20.4) = \frac{6\,800\,000}{20.4} + 400(20.4)^2$$

$$= 499\,800$$

$C(100) = 4\,063\,000.$

Therefore, the dimensions that will keep the cost to a minimum are 20.4 m by 40.8 m by 24.0 m.

10. Let the height of the cylinder be h cm, the radius r cm. Let the cost for the walls be $\$k$ and for the top $\$2k$.

$V = 1000 = \pi r^2 h$ or $h = \frac{1000}{\pi r^2}$

The cost $C = (2\pi r^2)(2k) + (2\pi rh)k$

or $C = 4\pi kr^2 + 2\pi kr\left(\frac{1000}{\pi r^2}\right)$

$$C(r) = 4\pi kr^2 + \frac{2000k}{r}, r \geq 0$$

$$C'(r) = 8\pi kr - \frac{2000k}{r^2}$$

Let $C'(r) = 0$, then $8\pi kr = \frac{2000k}{r^2}$

or $r^3 = \frac{2000}{8\pi}$

$r \doteq 4.3$

$$h = \frac{1000}{\pi(4.3)^2} = 17.2.$$

Since $r \geq 0$, minimum cost occurs when $r = 4.3$ cm and $h = 17.2$ cm.

11. a. Let the number of \$0.50 increase be n.

New price $= 10 + 0.5n$.

Number sold $= 200 - 7n$.

Revenue $R(n) = (10 + 0.5n)(200 - 7n)$

$= 2000 + 30n - 3.5n^2$

Profit $P(n) = R(n) - C(n)$

$= 2000 + 30n + 3.5n^2 - 6(200 - 7n)$

$= 800 + 72n - 3.5n^2$

$P'(n) = 72 - 7n$

Let $P'(n) = 0$,

$72 - 7n = 0, n \doteq 10.$

Price per cake = 10 + 5 = \$15

Number sold = 200 − 70 = 130

b. Since 200 − 165 = 35, it takes 5 price increases to reduce sales to 165 cakes.

New price is 10 + 0.5 × 5 = \$12.50.

The profit is 165 × 5 = \$825.

c. If you increase the price, the number sold will decrease. Profit in situation like this will increase for several price increases and then it will decrease because too many customers stop buying.

12. Let x be the base length and y be the height.

Top/bottom: $20/m^2

Sides: $30/m^2

$$4000\text{ cm}^3\left(\frac{1\text{ m}}{100\text{ cm}}\right)^3 = 0.004\text{ m}^3$$

$$0.004 = x^2y$$

$$y = \frac{0.004}{x^2}$$

$$A_{\text{Top}} + A_{\text{Bottom}} = x^2 + x^2 = 2x^2$$

$$4A_{\text{Side}} = 4xy$$

$$C = 20(2x^2) + 30(4xy)$$

$$C = 40x^2 + 120x\left(\frac{0.004}{x^2}\right)$$

$$C = 40x^2 + 0.48x^{-1}$$

$$C' = 80x - 0.48x^{-2}$$

Set $C' = 0$

$$0 = 80x - 0.48x^{-2}$$

$$80x^3 = 0.48$$

$$x^3 = 0.006$$

$$x \doteq 0.182$$

$$y = \frac{0.004}{0.182^2}$$

$$y \doteq 0.121$$

$$C'(1) = 79.52 > 0$$

$$C'(-1) = -80.48 < 0$$

maximum

The jewellery box should be

12.1 cm × 18.2 cm × 18.2 cm to minimize the cost of materials.

13. Let x be the number of price changes and R be the revenue.

$$R = (90 - x)(50 + 5x)$$

$$R' = 5(90 - x) - 1(50 + 5x)$$

Set $R' = 0$

$$0 = 5(90 - x) - 1(50 + 5x)$$

$$0 = 450 - 5x - 50 - 5x$$

$$0 = 400 - 10x$$

$$10x = 400$$

$$x = 40$$

Price = \$90 − \$40

Price = \$50

$$R'(0) = 400 > 0$$

$$R'(100) = -600 < 0$$

maximum: The price of the CD player should be \$50.

14. Let x be the number of price changes and R be the revenue.

$$R = (75 - 5x)(14\,000 + 800x),\ x \le 7.5$$

$$R' = 800(75 - 5x) + (-5)(14\,000 + 800x)$$

Set $R' = 0$

$$0 = 60\,000 - 4000x - 70\,000 - 4000x$$

$$10\,000 = -8000x$$

$$x = -1.25$$

Price = \$75 − \$5(−1.25)

Price = \$81.25

$$R'(-2) = 6000 > 0$$

$$R'(2) = -26\,000 < 0$$

maximum: The price of a ticket should be \$81.25.

15. $P(x) = (2000 - 5x)(1000x) - (15\,000\,000 + 1\,800\,000x + 75x^2)$

$$P(x) = 2\,000\,000x - 5000x^2 - 15\,000\,000 - 1\,800\,000x - 75x^2$$

$$P(x) = -5075x^2 + 200\,000x - 15\,000\,000$$

$$P'(x) = -10\,150x + 200\,000$$

Set $P'(x) = 0$

$$0 = -10\,150x + 200\,000$$

$$10\,150x = 200\,000$$

$$x \doteq 19.704$$

$$P'(0) = 200\,000 > 0$$

$$P'(20) = -3000 < 0$$

maximum: The computer manufacturer should sell 19 704 units to maximize profit.

16. $P(x) = R(x) - C(x)$

Marginal Revenue = $R'(x)$.

Marginal Cost = $C'(x)$.

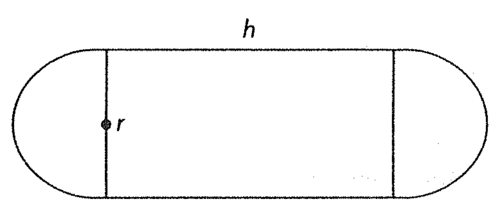
Now $P'(x) = R'(x) - C'(x)$.

The critical point occurs when $P'(x) = 0$.

If $R'(x) = C'(x)$, then $P'(x) = R'(x) - R'(x) = 0$.

Therefore, the instantaneous rate of change in profit is 0 when the marginal revenue equals the marginal cost.

17.



Label diagram as shown, Let cost of cylinder be $k/m³.

$V = 200$

$= \pi r^2 h + \frac{4}{3}\pi r^3$

Note: Surface Area = Total cost C

Cost $C = (2\pi rh)k + (4\pi r^2)2k$

But, $200 = \pi r^2 h + \frac{4}{8}\pi r^3$ or $600 = 3\pi r^2 h + 4\pi r^2$

Therefore, $h = \frac{600 - 4\pi r^3}{3\pi r^2}$.

$$C(r) = 2k\pi r\left(\frac{600 - 4\pi r^3}{3\pi r^2}\right)1 + 8k\pi r^2$$

$$= 2k\left(\frac{600 - 4\pi r^3}{3r}\right) + 8k\pi r^2$$

Since $h \leq 16$, $r \leq \left(\frac{600}{4\pi}\right)^{\frac{1}{3}}$ or $0 \leq r \leq 3.6$

$$C(r) = \frac{400k}{r} - \frac{8k\pi r^2}{3} + 3k\pi r^2$$

$$= \frac{400k}{r} + \frac{16k\pi r^2}{3}$$

$$C'(r) = -\frac{400k}{r^2} + \frac{32k\pi r}{3}$$

Let $C'(r) = 0$

$$\frac{400k}{r^2} = \frac{32k\pi r}{3}$$

$$\frac{50}{r^2} = \frac{4\pi r}{3}$$

$$4\pi r^3 = 150$$

$$r^3 = \frac{150}{4\pi}$$

$$r = 2.29$$

$$h \doteq 8.97 \text{ m}$$

Note: $C(0) \to \infty$

$C(2.3) \doteq 262.5k$

$C(3.6) \doteq 330.6k$

The minimum cost occurs when $r = 230$ cm and h is about 900 cm.

18. $C = 1.15 \times \frac{450}{8 - .1(s - 110)} + (35 + 15.5)\frac{450}{s}$

$$C = \frac{517.5}{-.1s + 19} + \frac{22\,725}{s}$$

$$C = \frac{517.5s - 2272.5s + 431\,775}{19s - .1s^2}$$

$$C = \frac{431\,775 - 1755s}{19s - .1s^2}$$

To find the value of s that minimizes C, we need to calculate the derivative of C.

$$C' = \frac{-1755(19s - .1s^2)}{(19s - .1s^2)^2} - \frac{(431\,775 - 1755s)(19 - .2s)}{(19s - .1s^2)^2}$$

$$C' = \frac{(-33\,345s + 175.5s^2)}{(19s - .1s^2)^2} - \frac{(8\,203\,725 - 119\,700s + 351s^2)}{(19s - .1s^2)^2}$$

$$C' = \frac{-175.5s^2 + 86\,355s - 8\,203\,725}{(19s - .1s^2)^2}$$

Let $C' = 0$:

$$\frac{-175.5s^2 + 86\,355s - 8\,203\,725}{(19s - .1s^2)^2} = 0$$

$$s = 128.4$$

There is a critical number at $s = 128.4$ km/h

s	s < 128.4	128.4	s > 128.4
C′(s)	−	0	+
Graph	Dec.	Local Min	Inc.

There is a local minimum for $s = 128.4$, so the cost is minimized for a speed of 128.4 km/h.

19. $v(r) = Ar^2(r_0 - r), 0 \leq r \leq r_0$

$v(r) = Ar_0 r^2 - Ar^3$

$v'(r) = 2Ar_0 r - 3Ar^2$

Let $v'(r) = 0$:

$2Ar_0 r - 3Ar^2 = 0$

$2r_0 r - 3r^2 = 0$

$r(2r_0 - 3r) = 0$

$r = 0$ or $r = \frac{2r_0}{3}$.

$v(0) = 0$

$$v\left(\frac{2r_0}{3}\right) = A\left(\frac{4}{9}r_0^2\right)\left(r_0 - \frac{2r_0}{3}\right)$$

$$= \frac{4}{27}r_0 A$$

$A(r_0) = 0$

The maximum velocity of air occurs when radius is $\frac{2r_0}{3}$.

Review Exercise, pp. 156–159

1. $f(x) = x^4 - \frac{1}{x^4}$

$= x^4 - x^{-4}$

$f'(x) = 4x^3 + 4x^{-5}$

$f''(x) = 12x^2 - 20x^{-6}$

2. $y = x^9 - 7x^3 + 2$

$\frac{dy}{dx} = 9x^8 - 21x^2$

$\frac{d^2y}{dx^2} = 72x^7 - 42x$

3. $s(t) = t^2 + 2(2t - 3)^{\frac{1}{2}}$

$v = s'(t) = 2t + \frac{1}{2}(2t - 3)^{-\frac{1}{2}}(2)$

$= 2t + (2t - 3)^{-\frac{1}{2}}$

$a = s''(t) = 2 - \frac{1}{2}(2t - 3)^{-\frac{3}{2}}(2)$

$= 2 - (2t - 3)^{-\frac{3}{2}}$

4. $s(t) = t - 7 + \frac{5}{t}$

$= t - 7 + 5t^{-1}$

$v(t) = 1 - 5t^{-2}$

$a(t) = 10t^{-3}$

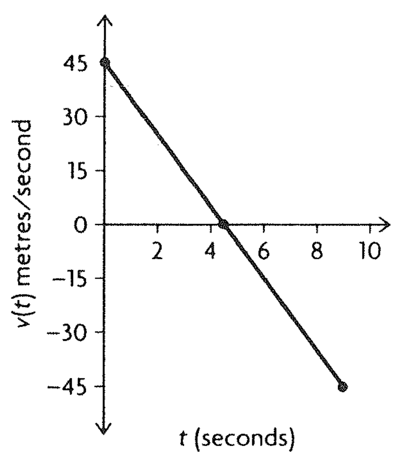
5. $s(t) = 45t - 5t^2$

$v(t) = 45 - 10t$

For $v(t) = 0, t = 4.5$.

t	$0 \le t < 4.5$	4.5	$t > 4.5$
$v(t)$	+	0	−

Therefore, the upward velocity is positive for $0 \le t < 4.5$ s, zero for $t = 4.5$ s, negative for $t > 4.5$ s.



6. a. $f(x) = 2x^3 - 9x^2$

$f'(x) = 6x^2 - 18x$

For max min, $f'(x) = 0$:

$6x(x - 3) = 0$

$x = 0$ or $x = 3$.

x	$f(x) = 2x^3 - 9x^2$	
−2	−52	min
0	0	max
3	−27	
4	−16	

The minimum value is −52.
The maximum value is 0.

b. $f(x) = 12x - x^3$

$f'(x) = 12 - 3x^2$

For max min, $f'(x) = 0$:

$3(4 - x^2) = 0$

$x = -2$ or $x = 2$

x	$f(x) = 12x - x^3$	
−3	−9	
−2	−16	
2	16	max
5	−65	min

c. $f(x) = 2x + \frac{18}{x}$

$f'(x) = 2 - 18x^{-2}$

For max min, $f'(x) = 0$:

$\frac{18}{x^2} = 2$

$x^2 = 9$

$x = \pm 3$.

x	$f(x) = 2x + \frac{18}{x}$
1	20
3	12
5	$10 + \frac{18}{5} = 13.6$

The minimum value is 12.
The maximum value is 20.

7. a. $s(t) = 62 - 16t + t^2$

$v(t) = -16 + 2t$

$s(0) = 62$

Therefore, the front of the car was 62 m from the stop sign.

b. When $v = 0, t = 8$,

$s(8) = 62 - 16(8) + (8)^2$

$= 62 - 128 + 64$

$= -2$

Yes, the car goes 2 m beyond the stop sign before stopping.

c. Stop signs are located two are more metres from an intersection. Since the car only went 2 m beyond the stop sign, it is unlikely the car would hit another vehicle travelling perpendicular.

8. $s(t) = 1 + 2t - \dfrac{8}{t^2 + 1}$

$v(t) = 2 + 8(t^2 + 1)^{-2}(2t) = 2 + \dfrac{16t}{(t^2 + 1)^2}$

$a(t) = 16(t^2 + 1)^{-2} + 16t(-2)(t^2 + 1)^{-3}2t$
$= 16(t^2 + 1)^{-2} - 64t^2(t^2 + 1)^{-3}$
$= 16(t^2 + 1)^{-3}[t^2 + 1 - 4t^2]$

For max min velocities, $a(t) = 0$:
$3t^2 = 1$
$t = \pm\dfrac{1}{\sqrt{6}}.$

t	$v(t) = 2 + \dfrac{16t}{(t^2 + 1)^2}$
0	2 min
$\dfrac{1}{\sqrt{3}}$	$2 + \dfrac{\frac{16}{\sqrt{3}}}{(\frac{1}{3} + 1)^2} = 2 + \dfrac{\frac{16\sqrt{3}}{3}}{\frac{16}{9}} = 2 + 3\sqrt{3}$ max
2	$2 + \dfrac{32}{25} = 3.28$

The minimum value is 2.

The maximum value is $2 + 3\sqrt{3}$.

9. $u(x) = 625x^{-1} + 15 + 0.01x$
$u'(x) = -625x^2 + 0.01$
For a minimum, $u'(x) = 0$
$x^2 = 62\,500$
$x = 250$

x	$u(x) = \dfrac{625}{x} + 0.01x$
1	625.01
250	2.5 + 2.5 = 5 min
500	$\dfrac{625}{500} + 5 = 6.25$

Therefore, 250 items should be manufactured to ensure unit waste is minimized.

10. a. $C(x) = 3x + 1000$

i. $C(400) = 1200 + 1000$
$= 2200$

ii. $\dfrac{2200}{400} = \$5.50$

iii. $C'(x) = 3$

The marginal cost when $x = 400$ and the cost of producing the 401st item are \$3.00.

b. $C(x) = 0.004x^2 + 40x + 8000$

i. $C(400) = 640 + 16\,000 + 8000$
$= 24\,640$

ii. $\dfrac{24\,640}{400} = \$61.60$

iii. $C'(x) = 0.008x + 40$
$C'(400) = 0.008(400) + 40$
$= 43.20$
$C'(401) = 0.008(401) + 40$
$= \$43.21$

The marginal cost when $x = 400$ is \$43.20, and the cost of producing the 401st item is \$43.21.

c. $C(x) = \sqrt{x} + 5000$

i. $C(400) = 20 + 5000$
$= \$5020$

ii. $C(400) = \dfrac{5020}{400}$
$= \$12.55$

iii. $C'(x) = \dfrac{1}{2}x^{-\frac{1}{2}}$
$= \dfrac{1}{2\sqrt{x}}$
$C'(400) = \dfrac{1}{40}$
$= \$0.025$
$\doteq \$0.03$
$C'(401) = \dfrac{1}{2\sqrt{401}}$
$= \$0.025$
$\doteq \$0.03$

The cost to produce the 401st item is \$0.03.

d. $C(x) = 100x^{\frac{1}{2}} + 5x + 700$

i. $C(400) = \dfrac{100}{20} + 2000 + 700$
$= \$2705$

ii. $C(400) = \dfrac{2750}{400}$
$= \$6.875$
$= \$6.88$

iii. $C'(x) = -50x^{-\frac{3}{2}} + 5$
$C'(400) = \dfrac{-50}{(20)^3} + 5$
$= 5.00625$
$= \$5.01$
$C'(401) = \$5.01$

The cost to produce the 401st item is \$5.01.

11. $C(x) = 0.004x^2 + 40x + 16\,000$
Average cost of producing x items is

$C(x) = \frac{C(x)}{x}$

$C(x) = 0.004x + 40 + \frac{16\,000}{x}$

To find the minimum average cost, we solve

$C'(x) = 0$

$0.004 - \frac{16\,000}{x^2} = 0$

$4x^2 - 16\,000\,000 = 0$

$x^2 = 4\,000\,000$

$x = 2000, x > 0$

From the graph, it can be seen that $x = 2000$ is a minimum. Therefore, a production level of 2000 items minimizes the average cost.

12. a. $s(t) = 3t^2 - 10$

$v(t) = 6t$

$v(3) = 18$

$v(3) > 0$, so the object is moving to the right.

$s(3) = 27 - 10 = 17$. The object is to the right of the starting point and moving to the right, so it is moving away from its starting point.

b. $s(t) = -t^3 + 4t^2 - 10$

$s(0) = -10$

Therefore, its starting position is at -10.

$s(3) = -27 + 36 - 10$

$= -1$

$v(t) = -3t^2 + 8t$

$v(3) = -27 + 24$

$= -3$

Since $s(3)$ and $v(3)$ are both negative, the object is moving away from the origin and towards its starting position.

13. $s = 27t^3 + \frac{16}{t} + 10, t > 0$

a. $v = 81t^2 - \frac{16}{t^2}$

$81t^2 - \frac{16}{t^2} = 0$

$81t^4 = 16$

$t^4 = \frac{16}{81}$

$t = \pm\frac{2}{3}$

$t > 0$

Therefore, $t = \frac{2}{3}$.

b. $a = \frac{dv}{dt} = 162t + \frac{32}{t^3}$

At $t = \frac{2}{3}$, $a = 162 \times \frac{2}{3} + \frac{32}{\frac{2}{3}}$

$= 216$

Since $a > 0$, the particle is accelerating.

14. Let the base be x cm by x cm and the height h cm.

Therefore, $x^2h = 10\,000$.

$A = x^2 + 4xh$

But $h = \frac{10\,000}{x^2}$.

$A(x) = x^2 + 4x\left(\frac{10\,000}{x^2}\right)$

$= x^2 + \frac{400\,000}{x}$, for $x \geq 5$

$A'(x) = 2x - \frac{400\,000}{x^2}$,

Let $A'(x) = 0$, then $2x = \frac{400\,000}{x^2}$

$x^3 = 200\,000$

$x = 27.14$.

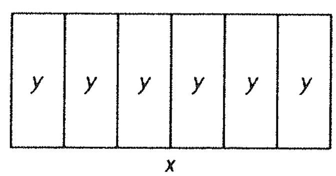
Using the max min Algorithm,

$A(5) = 25 + 80\,000 = 80\,025$

$A(27.14) \doteq 15\,475$

The dimensions of a box of minimum area is 27.14 cm for the base and height 13.57 cm.

15. Let the length be x and the width y.



$P = 2x + 6y$ and $xy = 12\,000$ or $y = \frac{12\,000}{x}$

$P(x) = 2x + 6 \times \frac{12\,000}{x}$

$P(x) = 2x + \frac{72\,000}{x}$, $10 \leq x \leq 1200(5 \times 240)$

$A'(x) = 2 - \frac{72\,000}{x^2}$

Let $A'(x) = 0$,

$2x^2 = 72\,000$

$x^2 = 36\,000$

$x \doteq 190$.

Using max min Algorithm,

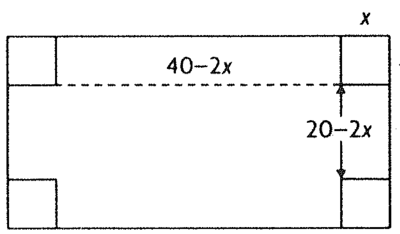
$A(10) = 20 + 7200 = 7220\text{ m}^2$

$A(190) \doteq 759\text{ m}^2$

$A(1200) = 1\,440\,060$

The dimensions for the minimum amount of fencing is a length of 190 m by a width of approximately 63 m.

16.



Let the width be w and the length $2w$.

Then, $2w^2 = 800$

$w^2 = 400$

$w = 20,\ w > 0.$

Let the corner cuts be x cm by x cm. The dimensions of the box are shown. The volume is

$$V(x) = x(40 - 2x)(20 - 2x)$$
$$= 4x^3 - 120x^2 - 800x,\ 0 \le x \le 10$$
$$V'(x) = 12x^2 - 240x - 800$$

Let $V'(x) = 0$:

$12x^2 - 240x - 800 = 0$

$3x^2 - 60x - 200 = 0$

$$x = \frac{60 \pm \sqrt{3600 - 2400}}{6}$$

$x \doteq 15.8$ or $x = 4.2$, but $x \le 10$.

Using max min Algorithm,

$V(0) = 0$

$V(4.2) = 1540\ \text{cm}^2$

$V(10) = 0.$

Therefore, the base is

$40 - 2 \times 4.2 = 31.6$

by $20 - 2 \times 4.2 = 11.6$

The dimensions are 31.6 cm by 11.6 cm by 4.2 cm.

17. Let the radius be r cm and the height h cm.

$V = \pi r^2 h = 500$

$A = 2\pi r^2 + 2\pi rh$

Since $h = \dfrac{500}{\pi r^2}$, $6 \le h \le 15$

$$A(r) = 2\pi r^2 + 2\pi r\left(\frac{500}{\pi r^2}\right)$$
$$= 2\pi r^2 + \frac{1000}{r} \text{ for } 2 \le r \le 5$$
$$A'(r) = 4\pi r - \frac{1000}{r^2}.$$

Let $A'(r) = 0$, then $4\pi r^3 = 1000$,

$$r^3 = \frac{1000}{4\pi}$$

$r \doteq 4.3.$

Using max min Algorithm,

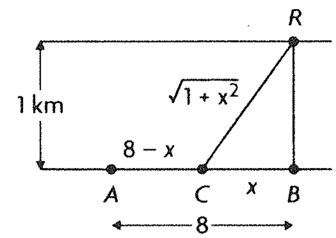
$A(2) \doteq 550$

$A(4.3) \doteq 349$

$A(5) \doteq 357$

For a minimum amount of material, the can should be constructed with a radius of 4.3 cm and a height of 8.6 cm.

18.



Let x be the distance CB, and $8 - x$ the distance AC.

Let the cost on land be \$$k$ and under water \$$1.6k$.

The cost $C(x) = k(8 - x) + 1.6k\sqrt{1 + x^2}$, $0 \le x \le 8.$

$$C'(x) = -k + 1.6k \times \frac{1}{2}(1 + x^2)^{-\frac{1}{2}}(2x)$$
$$= -k + \frac{1.6kx}{\sqrt{1 + x^2}}$$

Let $C'(x) = 0$,

$$-k + \frac{1.6kx}{\sqrt{1 + k^2}} = 0$$
$$\frac{1.6x}{\sqrt{1 + k^2}} = 1$$
$$1.6x = \sqrt{1 + x^2}$$
$$2.56x^2 = 1 + x^2$$
$$1.56x^2 = 1$$
$$x^2 \doteq 0.64$$
$$x = 0.8,\ x > 0$$

Using max min Algorithm,

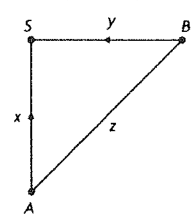
$A(0) = 9.6k$

$A(0.8) = k(8 - 0.8) + 1.6k\sqrt{1 + (0.8)^2} = 9.25k$

$A(8) = 12.9k$

The best way to cross the river is to run the pipe $8 - 0.8$ or 7.2 km along the river shore and then cross diagonally to the refinery.

19.



Let y represent the distance the westbound train is from the station and x the distance of the

northbound train from the station S. Let t represent time after 10:00.
Then $x = 100t$, $y = (120 - 120t)$
Let the distance AB be z.

$$z = \sqrt{(100t)^2 + (120 - 120t)^2}, 0 \leq t \leq 1$$

$$\frac{dz}{dt} = \frac{1}{2}[(100t)^2 + (120 - 120t)^2]^{-\frac{1}{2}}$$

$$\times [2 \times 100 \times 100t - 2 \times 120 \times (120(1 - t))]$$

Let $\frac{dz}{dt} = 0$, that is

$$\frac{2 \times 100 \times 100t - 2 \times 120 \times 120(1 - t)}{2\sqrt{(100t)^2 + (120 - 120t)^2}} = 0$$

or $20\,000t = 28\,800(1 - t)$

$48\,800t = 288\,000$

$t = \frac{288}{488} \doteq 0.59$ h or 35.4 min.

When $t = 0$, $z = 120$.

$t = 0.59$

$$z = \sqrt{(100 \times 0.59)^2 + (120 - 120 \times 0.59)^2}$$
$$= 76.8 \text{ km}$$

$t = 1$, $z = 100$

The closest distance between trains is 76.8 km and occurs at 10:35.

20. Let the number of price increases be n.
New selling price $= 100 + 2n$.
Number sold $= 120 - n$.
Profit $=$ Revenue $-$ Cost

$P(n) = (100 + 2n)(120 - n) - 70(120 - n)$,
$0 \leq n \leq 120$
$= 3600 + 210n - 2n^2$

$P'(n) = 210 - 4n$

Let $P'(n) = 0$

$210 - 4n = 0$

$n = 52.5$.

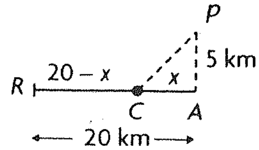
Therefore, $n = 52$ or 53.

Using max min Algorithm,

$P(0) = 3600$
$P(52) = 9112$
$P(53) = 9112$
$P(120) = 0$

The maximum profit occurs when the portable MP3 are sold at \$204 for 68 and at \$206 for 67 portable MP3.

21.



Let x represent the distance AC.

Then, $RC = 20 - x$ and 4.

$PC = \sqrt{25 + x^2}$

The cost:

$C(x) = 100\,000\sqrt{25 + x^2} + 75\,000(20 - x)$,
$0 \leq x \leq 20$

$$C'(x) = 100\,000 \times \frac{1}{2}(25 + x^2)^{-\frac{1}{2}}(2x) - 75\,000.$$

Let $C'(x) = 0$,

$$\frac{100\,000x}{\sqrt{25 + x^2}} - 75\,000 = 0$$

$$4x = 3\sqrt{25 + x^2}$$
$$16x^2 = 9(25 + x^2)$$
$$7x^2 = 225$$
$$x^2 \doteq 32$$
$$x \doteq 5.7.$$

Using max min Algorithm,

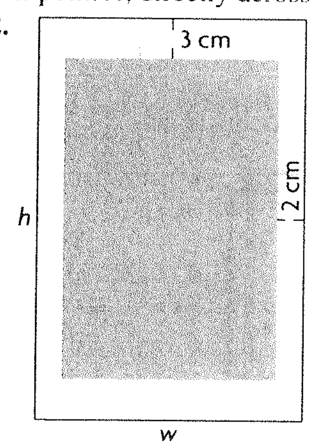
$A(0) = 100\,000\sqrt{25} + 75\,000(20) = 2\,000\,000$

$A(5.7) = 100\,000\sqrt{25 + 5.7^2} + 75\,000(20 - 5.7)$
$= 1\,830\,721.60$

$A(20) = 2\,061\,552.81$.

The minimum cost is \$1 830 722 and occurs when the pipeline meets the shore at a point C, 5.7 km from point A, directly across from P.

22.



$$A = hw$$
$$81 = (h - 6)(w - 4)$$
$$\frac{81}{h - 6} = w - 4$$
$$\frac{81}{h - 6} + 4 = w$$
$$\frac{81 + 4(h - 6)}{h - 6} = w$$
$$\frac{4h + 57}{h - 6} = w$$

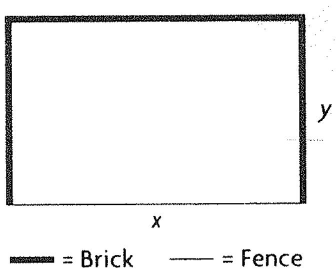
Substitute for w in the area equation and differentiate:

$$A = (h)\frac{4h + 57}{h - 6}$$
$$A = \frac{4h^2 + 57h}{h - 6}$$
$$A' = \frac{(8h + 57)(h - 6) - (4h^2 + 57h)}{(h - 6)^2}$$
$$A' = \frac{8h^2 + 9h - 342 - 4h^2 - 57h}{(h - 6)^2}$$
$$A' = \frac{4h^2 - 48h - 342}{(h - 6)^2}$$
Let $A' = 0$:
$$\frac{4h^2 - 48h - 342}{(h - 6)^2} = 0$$
Therefore, $4h^2 - 48h - 342 = 0$
Using the quadratic formula, $h = 17.02$ cm

h	$t < 17.02$	17.02	$t > 17.02$
A′(h)	−	0	+
Graph	Dec.	Local Min	Inc.

There is a local minimum at $h = 17.02$ cm, so that is the minimizing height.
$$81 = (h - 6)(w - 4)$$
$$81 = 11.02(w - 4)$$
$$7.35 = w - 4$$
$$w = 11.35 \text{ cm}$$
The dimensions of the page should be $11.35 \text{ cm} \times 17.02 \text{ cm}$.

23.

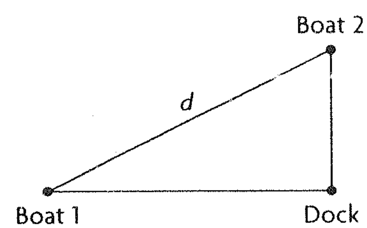


$$C = (192 + 48)x + 192(2y)$$
$$C = 240x + 284y$$
$$1000 = xy$$
$$\frac{1000}{y} = x$$
Substitute $\frac{1000}{y}$ for y in the cost equation and differentiate to find the minimizing value for x:
$$C = 240\frac{1000}{y} + 284y$$
$$C = \frac{240\,000}{y} + 284y$$
$$C' = \frac{-240\,000}{y^2} + 284$$
$$C' = \frac{284y^2 - 240\,000}{y^2}$$
Let $C' = 0$:
$$\frac{284y^2 - 240\,000}{y^2} = 0$$
Therefore $284y^2 - 240\,000 = 0$
$$284y^2 = 240\,000$$
$$y = 29.1 \text{ m}$$

y	$y < 29.1$	29.1	$y > 29.1$
C′(y)	−	0	+
Graph	Dec.	Local Min	Inc.

There is a local minimum at $y = 29.1$ m, so that is the minimizing value. To find x, use the equation
$$\frac{1000}{y} = x$$
$$\frac{1000}{29.1} = x$$
$$x = 34.4 \text{ m}$$
The fence and the side opposite it should be 34.4 m, and the other two sides should be 29.1 m.

24.

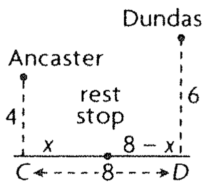


The distance between the boats is the hypotenuse of a right triangle. One side of the triangle is the distance from the first boat to the dock and the other side is the distance from the second boat to the dock. The distance is given by the equation
$d(t) = \sqrt{(15t)^2 + (12 - 12t)^2}$ where t is hours after 2:00
$$d(t) = \sqrt{369t^2 - 288t + 144}$$
To find the time that minimizes the distance, calculate the derivative and find the critical numbers:
$$d'(t) = \frac{738t - 288}{2\sqrt{81t^2 - 48t + 144}}$$
Let $d'(t) = 0$:
$$\frac{738t - 288}{2\sqrt{81t^2 - 48t + 144}} = 0$$
Therefore, $738t - 288 = 0$
$$738t = 288$$
$$t = .39 \text{ hours}$$

t	$t < .39$	.39	$t > .39$
$d'(t)$	$-$	0	$+$
Graph	Dec.	Local Min	Inc.

There is a local minimum at $t = .39$ hours, so the ships were closest together at 2:23.

25.



Let the distance from C to the rest stop be x and so the distance from the rest stop to D is $8 - x$, as shown. The distance from Ancaster to the rest stop is therefore

$\sqrt{4^2 + x^2} = \sqrt{16 + x^2}$, and the distance from the rest stop to Dundas is

$$\sqrt{6^2 + (8 - x)^2} = \sqrt{36 + 64 - 16x + x^2}$$
$$= \sqrt{100 - 16x + x^2}$$

So the total length of the trails is

$L = \sqrt{16 + x^2} + \sqrt{100 - 16x + x^2}$

The minimum cost can be found by expressing L as a function of x and examining its derivative to find critical points.

$L(x) = \sqrt{16 + x^2} + \sqrt{100 - 16x + x^2}$, which is defined for $0 \le x \le 8$

$$L'(x) = \frac{2x}{2\sqrt{16 + x^2}} + \frac{2x - 16}{2\sqrt{100 - 16x + x^2}}$$
$$= \frac{x\sqrt{100 - 16x + x^2} + (x - 8)\sqrt{16 + x^2}}{\sqrt{(16 + x^2)(100 - 16x + x^2)}}$$

The critical points of $A(r)$ can be found by setting $L'(x) = 0$:

$$x\sqrt{100 - 16x + x^2} + (x - 8)\sqrt{16 + x^2} = 0$$
$$x^2(100 - 16x + x^2) = (x^2 - 16x + 64)(16 + x^2)$$
$$100x^2 - 16x^3 + x^4 = x^4 - 16x^3 + 64x^2 + 16x^2 - 256x + 1024$$
$$20x^2 + 256x - 1024 = 0$$
$$4(5x - 16)(x + 16) = 0$$

So $x = 3.2$ and $x = -16$ are the critical points of the function. Only the positive root is within the interval of interest, however. The minimum total length therefore occurs at this point or at one of the endpoints of the interval:

$$L(0) = \sqrt{16 + 0^2} + \sqrt{100 - 16(0) + 0^2} = 14$$
$$L(3.2) = \sqrt{16 + 3.2^2} + \sqrt{100 - 16(3.2) + 3.2^2}$$
$$\doteq 12.8$$
$$L(8) = \sqrt{16 + 8^2} + \sqrt{100 - 16(8) + 8^2} \doteq 14.9$$

So the rest stop should be built 3.2 km from point C.

26. a. $f(x) = x^2 - 2x + 6,\ -1 \le x \le 7$

$f'(x) = 2x - 2$

Set $f'(x) = 0$

$0 = 2x - 2$

$x = 1$

$f(-1) = (-1)^2 - 2(-1) + 6$

$f(-1) = 1 + 2 + 6$

$f(-1) = 9$

$f(7) = (7)^2 - 2(7) + 6$

$f(7) = 49 - 14 + 6$

$f(7) = 41$

$f(1) = 1^2 - 2(1) + 6$

$f(1) = 1 - 2 + 6$

$f(1) = 5$

Absolute Maximum: $f(7) = 41$

Absolute Minimum: $f(1) = 5$

b. $f(x) = x^3 + x^2,\ -3 \le x \le 3$

$f'(x) = 3x^2 + 2x$

Set $f'(x) = 0$

$0 = 3x^2 + 2x$

$0 = x(3x + 2)$

$x = -\frac{2}{3}$ or $x = 0$

$f(-3) = (-3)^3 + (-3)^2$

$f(-3) = -27 + 9$

$f(-3) = -18$

$f\left(-\frac{2}{3}\right) = \left(-\frac{2}{3}\right)^3 + \left(-\frac{2}{3}\right)^2$

$f\left(-\frac{2}{3}\right) = -\frac{8}{27} + \frac{4}{9}$

$f\left(-\frac{2}{3}\right) = \frac{4}{27}$

$f(0) = (0)^3 + (0)^2$

$f(0) = 0$

$f(3) = (3)^3 + (3)^2$

$f(3) = 27 + 9$

$f(3) = 36$

Absolute Maximum: $f(3) = 36$

Absolute Minimum: $f(-3) = -18$

c. $f(x) = x^3 - 12x + 2,\ -5 \le x \le 5$

$f'(x) = 3x^2 - 12$

Set $f'(x) = 0$

$0 = 3x^2 - 12$

$x^2 = 4$

$x = -2$ or $x = 2$

$f(-5) = (-5)^3 - 12(-5) + 2$

$f(-5) = -125 + 60 + 2$

$f(-5) = -63$

$f(2) = (2)^3 - 12(2) + 2$

$f(2) = 8 - 24 + 2$

$f(2) = -14$

$f(-2) = (-2)^3 - 12(-2) + 2$

$f(-2) = -8 + 24 + 2$

$f(-2) = 18$

$f(5) = (5)^3 - 12(5) + 2$

$f(5) = 125 - 60 + 2$

$f(5) = 67$

Absolute Maximum: $f(5) = 67$

Absolute Minimum: $f(-5) = -63$

d. $f(x) = 3x^5 - 5x^3, -2 \le x \le 4$

$f'(x) = 15x^4 - 15x^2$

Set $f'(x) = 0$

$0 = 15x^4 - 15x^2$

$0 = 15x^2(x^2 - 1)$

$0 = 15x^2(x - 1)(x + 1)$

$x = -1$ or $x = 0$ or $x = 1$

$f(-2) = 3(-2)^5 - 5(-2)^3$

$f(-2) = -96 + 40$

$f(-2) = -56$

$f(0) = 3(0)^5 + 5(0)^3$

$f(0) = 0$

Note: (0, 0) is not a maximum or a minimum

$f(4) = 3(4)^5 - 5(4)^3$

$f(4) = 3072 - 320$

$f(4) = 2752$

$f(-1) = 3(-1)^5 - 5(-1)^3$

$f(-1) = -3 + 5$

$f(-1) = 2$

$f(1) = 3(1)^5 - 5(1)^3$

$f(1) = 3 - 5$

$f(1) = -2$

Absolute Maximum: $f(4) = 2752$

Absolute Minimum: $f(-2) = -56$

27. a. $s(t) = 20t - 0.3t^3$

$s'(t) = 20 - 0.9t^2$

The car stops when $s'(t) = 0$.

$20 - 0.9t^2 = 0$

$0.9t^2 = 20$

$t = \sqrt{\frac{20}{0.9}}$

$t \doteq 4.714$

(-4.714 is inadmissible)

$s(4.714) = 20(4.714) - 0.3(4.714)^3$

$\doteq 62.9$ m

b. From the solution to a., the stopping time is about 4.7 s.

c. $s''(t) = -1.8t$

$s''(2) = -1.8(2)$

$= -3.6 \text{ m/s}^2$

The deceleration is 3.6 m/s^2.

28. a. $f'(x) = \frac{d}{dx}(5x^3 - x)$

$= 15x^2 - 1$

$f''(x) = \frac{d}{dx}(15x^2 - 1)$

$= 30x$

So $f''(2) = 30(2) = 60$

b. $f'(x) = \frac{d}{dx}(-2x^{-3} + x^2)$

$= 6x^{-4} + 2x$

$f''(x) = \frac{d}{dx}(6x^{-4} + 2x)$

$= -24x^{-5} + 2$

So $f''(-1) = -24(-1)^{-5} + 2 = 26$

c. $f'(x) = \frac{d}{dx}(4x - 1)^4$

$= 4(4x - 1)^3(4)$

$= 16(4x - 1)^3$

$f''(x) = \frac{d}{dx}(16(4x - 1)^3)$

$= 16(3)(4x - 1)^2(4)$

$= 192(4x - 1)^2$

So $f''(0) = 192(4(0) - 1)^2 = 192$

d. $f'(x) = \frac{d}{dx}\left(\frac{2x}{x - 5}\right)$

$= \frac{(x - 5)(2) - (2x)(1)}{(x - 5)^2}$

$= \frac{-10}{(x - 5)^2}$

$f''(x) = \frac{d}{dx}\left(\frac{-10}{(x - 5)^2}\right)$

$= \frac{(x - 5)^2(0) - (-10)(2(x - 5))}{(x - 5)^4}$

$$= \frac{20}{(x-5)^3}$$

So $f''(1) = \frac{20}{(1-5)^3} = -\frac{5}{16}$

e. $f(x)$ can be rewritten as $f(x) = (x+5)^{\frac{1}{2}}$. Then

$$f'(x) = \frac{d}{dx}\left((x+5)^{\frac{1}{2}}\right)$$
$$= \frac{1}{2}(x+5)^{-\frac{1}{2}}$$
$$f''(x) = \frac{d}{dx}\left(\frac{1}{2}(x+5)^{-\frac{1}{2}}\right)$$
$$= \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)(x+5)^{-\frac{3}{2}}$$
$$= -\frac{1}{4}(x+5)^{-\frac{3}{2}}$$

So $f''(4) = -\frac{1}{4}(4+5)^{-\frac{3}{2}} = -\frac{1}{108}$

f. $f(x)$ can be rewritten as $f(x) = x^{\frac{2}{3}}$. Then

$$f'(x) = \frac{d}{dx}\left(x^{\frac{2}{3}}\right)$$
$$= \left(\frac{2}{3}\right)x^{-\frac{1}{3}}$$
$$f''(x) = \frac{d}{dx}\left(\left(\frac{2}{3}\right)x^{-\frac{1}{3}}\right)$$
$$= \left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)x^{-\frac{4}{3}}$$
$$= -\left(\frac{2}{9}\right)x^{-\frac{4}{3}}$$

So $f''(8) = -\left(\frac{2}{9}\right)(8)^{-\frac{4}{3}} = -\frac{1}{72}$

29. a. $s(t) = \frac{2t}{t+3}$

$$s'(t) = \frac{(t+3)(2) - 2t(1)}{(t+3)^2}$$
$$= \frac{2t + 6 - 2t}{(t+3)^2}$$
$$= \frac{6}{(t+3)^2}$$
$$s''(t) = \frac{(t+3)^2(0) - 6(2(t+3)+1)}{(t+3)^4}$$
$$= \frac{-6(2t+6)}{(t+3)^4}$$
$$= \frac{-12(t+3)}{(t+3)^4}$$
$$= \frac{-12}{(t+3)^3}$$
$$s(3) = \frac{2(3)}{3+3}$$
$$= \frac{6}{6}$$
$$= 1$$
$$s'(3) = \frac{6}{(3+3)^2}$$
$$= \frac{6}{36}$$
$$= \frac{1}{6}$$
$$s''(3) = \frac{-12}{(3+3)^3}$$
$$= \frac{-12}{216}$$
$$= -\frac{1}{18}$$

At $t = 3$, position is 1, velocity is $\frac{1}{6}$, acceleration is $-\frac{1}{18}$, and speed is $\frac{1}{6}$.

b. $s(t) = t + \frac{5}{t+2}$

$$s'(t) = 1 + \frac{(t+2)(0) - 5(1)}{(t+2)^2}$$
$$= 1 - \frac{5}{(t+2)^2}$$
$$s''(t) = 0 - \frac{(t+2)^2(0) - 5[2(t+2)(1)]}{(t+2)^4}$$
$$= \frac{10(t+2)}{(t+2)^4}$$
$$= \frac{10}{(t+2)^3}$$
$$s(1) = 1 + \frac{5}{1+2}$$
$$= 1 + \frac{5}{3}$$
$$= \frac{8}{3}$$
$$s'(1) = 1 - \frac{5}{(1+2)^2}$$
$$= 1 - \frac{5}{9}$$
$$= \frac{4}{9}$$

$$s''(1) = \frac{10}{(1 + 2)^3}$$
$$= \frac{10}{27}$$

At $t = 3$, position is $\frac{8}{3}$, velocity is $\frac{4}{9}$, acceleration is $\frac{10}{27}$, and speed is $\frac{4}{9}$.

30. a. $s(t) = (t^2 + t)^{\frac{2}{3}}, t \geq 0$

$$v(t) = \frac{2}{3}(t^2 + t)^{-\frac{1}{3}}(2t + 1)$$

$a(t)$

$$= \frac{2}{3}\left[-\frac{1}{3}(t^2 + t)^{-\frac{4}{3}}(2t + 1)(2t + 1) + 2(t^2 + 1)^{-\frac{1}{3}}\right]$$
$$= \frac{2}{3}\left(-\frac{1}{3}\right)(t^2 + t)^{-\frac{4}{3}}[(2t + 1)^2 - 6(t^2 + 1)]$$
$$= -\frac{2}{9}(t^2 + t)^{-\frac{4}{3}}(4t^2 + 4t + 1 - 6t^2 - 6t)$$
$$= \frac{2}{9}(t^2 + t)^{-\frac{4}{3}}(2t^2 + 2t - 1)$$

b. $$v_{avg} = \frac{s(5) - s(0)}{5 - 0}$$
$$= \frac{(5^2 + 5)^{\frac{2}{3}} - (0^2 + 0)^{\frac{2}{3}}}{5}$$
$$= \frac{30^{\frac{2}{3}} - 0}{5}$$
$$\doteq 1.931$$

The average velocity is approximately 1.931 m/s.

c. $$v(5) = \frac{2}{3}(5^2 + 5)^{-\frac{1}{3}}(2(5) + 1)$$
$$= \frac{2}{3}(30)^{-\frac{1}{3}}(11)$$
$$\doteq 2.360$$

The velocity at 5 s is approximately 2.36 m/s.

d. Average acceleration $= \frac{v(5) - v(0)}{5 - 0}$ which is undefined because $v(0)$ is undefined.

e. $$a(5) = \frac{2}{9}(5^2 + 5)^{-\frac{4}{3}}(2(5)^2 + 2(5) \pm 1)$$
$$= \frac{2}{9}\left(30^{-\frac{4}{3}}\right)(59)$$
$$\doteq 0.141$$

The acceleration at 5 s is approximately 0.141 m/s^2.

Chapter 3 Test, p. 160

1. a. $y = 7x^2 - 9x + 22$
$y' = 14x - 9$
$y'' = 14$

b. $f(x) = -9x^5 - 4x^3 + 6x - 12$
$f'(x) = -45x^4 - 12x^2 + 6$
$f''(x) = -180x^3 - 24x$

c. $y = 5x^{-3} + 10x^3$
$y' = -15x^{-4} + 30x^2$
$y'' = 60x^{-5} + 60x$

d. $f(x) = (4x - 8)^3$
$f'(x) = 3(4x - 8)^2(4)$
$= 12(4x - 8)^2$
$f''(x) = 24(4x - 8)(4)$
$= 96(4x - 8)$

2. a. $s(t) = -3t^3 + 5t^2 - 6t$
$v(t) = -9t^2 + 10t - 6$
$v(3) = -9(9) + 30 - 6$
$= -57$
$a(t) = -18t + 10$
$a(3) = -18(3) + 10$
$= -44$

b. $s(t) = (2t - 5)^3$
$v(t) = 3(2t - 5)^2(2)$
$= 6(2t - 5)^2$
$v(2) = 6(4 - 5)^2$
$= 6$
$a(t) = 12(2t - 5)(2)$
$= 24(2t - 5)$
$a(2) = 24(4 - 5)$
$= -24$

3. a. $s(t) = t^2 - 3t + 2$
$v(t) = 2t - 3$
$a(t) = 2$

b. $2t - 3 = 0$
$t = 1.5$ s
$s(1.5) = 1.5^2 - 3$
$(1.5) + 2 = -0.25$

c. $t^2 - 3t + 2 = 0$
$(t - 1)(t - 2) = 0$
$t = 1$ or $t = 2$
$|v(1)| = |-1|$
$= 1$
$|v(2)| = |1|$
$= 1$

The speed is 1 m/s when the position is 0.

d. The object moves to the left when $v(t) < 0$.
$2t - 3 < 0$
$t < 1.5$

The object moves to the left between $t = 0$ s and $t = 1.5$ s.

e. $v(5) = 10 - 3 = 7$ m/s
$v(2) = 4 - 3 = 1$ m/s

average velocity $= \dfrac{7 - 1}{5 - 2}$
$= 2$ m/s^2

4. a. $f(x) = x^3 - 12x + 2$
$f'(x) = 3x^2 - 12x$
$3x^2 - 12x = 0$
$3x(x - 4) = 0$
$x = 0$ or $x = 4$

Test the endpoints and the values that make the derivative 0.

$f(-5) = -125 + 60 + 2 = -63$ min
$f(0) = 2$
$f(4) = 64 - 48 + 2 = 18$
$f(5) = 125 - 60 + 2 = 67$ max

b. $f(x) = x + \dfrac{9}{x}$
$= x + 9x^{-1}$
$f'(x) = 1 - 9x^{-2}$
$1 - 9x^{-2} = 0$
$1 - \dfrac{9}{x^2} = 0$
$\dfrac{x^2 - 9}{x^2} = 0$
$x^2 - 9 = 0$
$x = \pm 3$

$x = -3$ is not in the given interval.

$f(1) = 1 + 9 = 10$ max
$f(3) = 3 + 3 = 6$ min
$f(6) = 6 + 1.5 = 7.5$

5. a. $h(t) = -4.9t^2 + 21t + 0.45$
$h'(t) = -9.8t + 21$

Set $h'(t) = 0$ and solve for t.

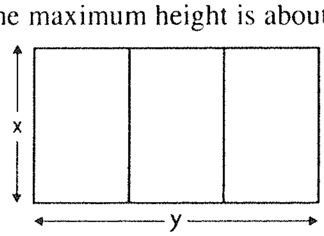
$-9.8t + 21 = 0$
$9.8t = 21$
$t \doteq 2.1$ s

The graph has a max or min at $t = 2.1$ s. Since the equation represents a parabola, and the lead coefficient is negative, the value must be a maximum.

b. $h(2.1) = -4.9(2.1)^2 + 21(2.1) + 0.45$
$\doteq 22.9$

The maximum height is about 22.9 m.

6.



Let x represent the width of the field in m, $x > 0$.
Let y represent the length of the field in m.

$4x + 2y = 2000$ ①
$A = xy$ ②

From ①: $y = 1000 - 2x$. Restriction $0 < x < 500$

Substitute into ②:

$A(x) = x(1000 - 2x)$
$= 1000x - 2x^2$
$A'(x) = 1000 - 4x$.

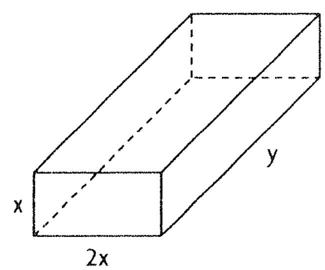
For a max min, $A'(x) = 0$, $x = 250$

x	$A(x) = x(1000 - 2x)$
0	$\lim_{x \to 0^+} A(x) = 0$
250	$A(250) = 125\,000$ max
1000	$\lim_{x \to 1000} A(x) = 0$

$x = 250$ and $y = 500$.

Therefore, each paddock is 250 m in width and $\frac{500}{3}$ m in length.

7.



Let x represent the height.
Let $2x$ represent the width.
Let y represent the length.

Volume $10\,000 = 2x^2y$

Cost:

$C = 0.02(2x)y + 2(0.05)(2x^2)$
$+ 2(0.05)(xy) + 0.1(2xy)$
$= 0.04xy + 0.2x^2 + 0.1xy + 0.2xy$
$= 0.34xy + 0.2x^2$

But $y = \dfrac{10\,000}{2x^2} = \dfrac{5000}{x^2}$.

Therefore, $C(x) = 0.34x\left(\dfrac{5000}{x^2}\right) + 0.2x^2$
$= \dfrac{1700}{x} + 0.2x^2, x \geq 0$
$C'(x) = \dfrac{-1700}{x^2} + 0.4x$.

Let $C'(x) = 0$:

$$\frac{-1700}{x^2} + 0.4x = 0$$

$$0.4x^3 = 1700$$

$$x^3 = 4250$$

$$x \doteq 16.2.$$

Using max min Algorithm,

$C(0) \to \infty$

$$C(16.2) = \frac{1700}{16.2} + 0.2(16.2)^2 = 157.4.$$

Minimum when $x = 16.2$, $2x = 32.4$ and $y = 19.0$.
The required dimensions are 162 mm by 324 mm by 190 mm.

8. Let x = the number of \$100 increases, $x \geq 0$.
The number of units rented will be $50 - 10x$.
The rent per unit will be $850 + 100x$.

$$R(x) = (850 + 100x)(50 - 10x)$$

$$R'(x) = (850 + 100x)(-10) + (50 - 10x)(100)$$

$$= -8500 - 1000x + 5000 - 1000x$$

$$= -2000x - 3500$$

Set $R'(x) = 0$

$0 = -3500 - 2000x$

$2000x = -3500$

$x = -1.75$ but $x \geq 0$

To maximize revenue the landlord should not increase rent. The residents should continue to pay \$850/month.