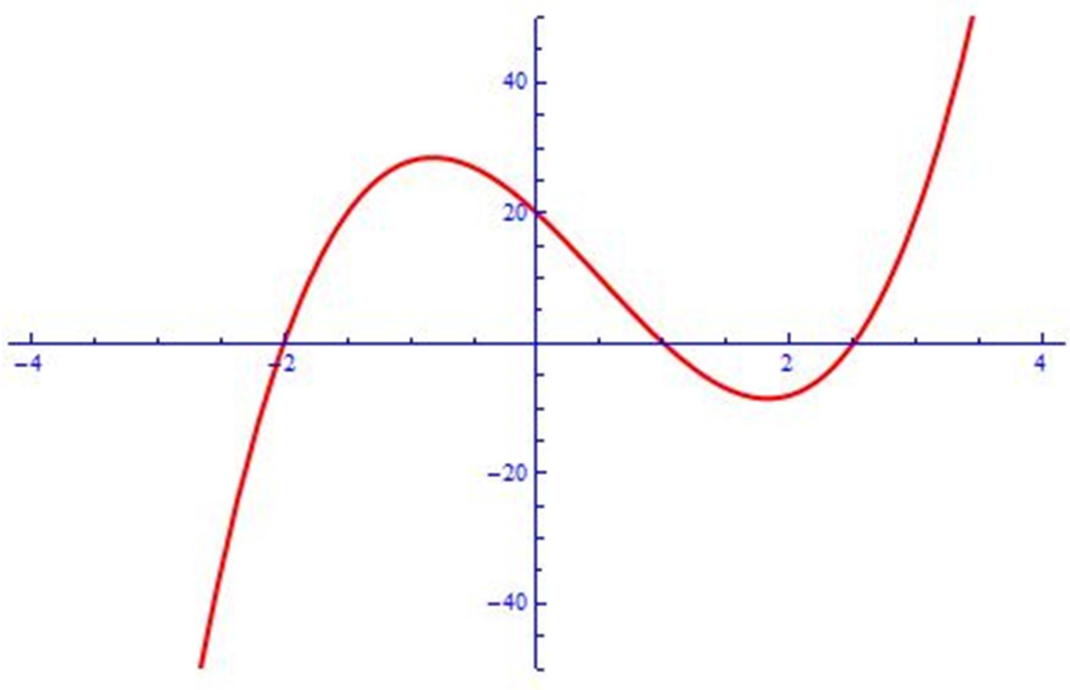
4.4 Concavity and Points of Inflection

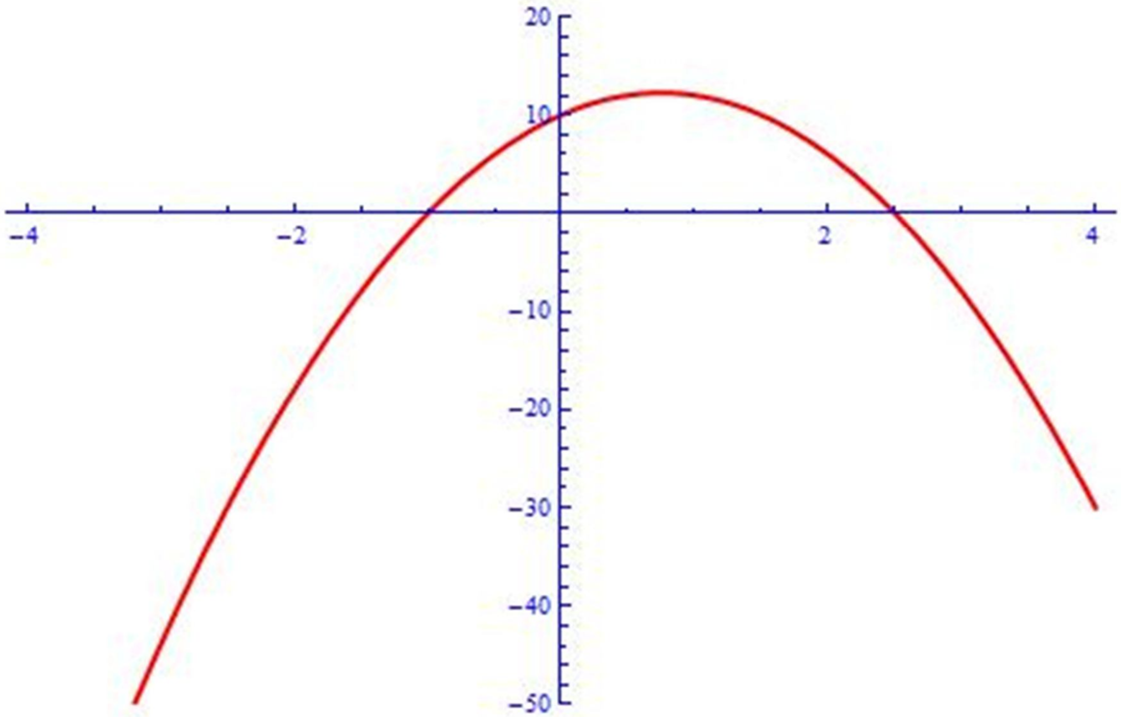
Recall that concave means to bend "away" from the perspective of observation.

Consider the sketch of the graph of $f(x)$



Q. How can we determine when an algebraic representation of a function is concave up or concave down?

Consider the Picture



Second Derivative Test

If $x = c$ is a critical value of some **twice differentiable** function $f(x)$ $\left(\text{i.e. } f'(c) = 0\right)$, then

If $f''(c) > 0$, $x = c$

If $f''(c) < 0$, $x = c$

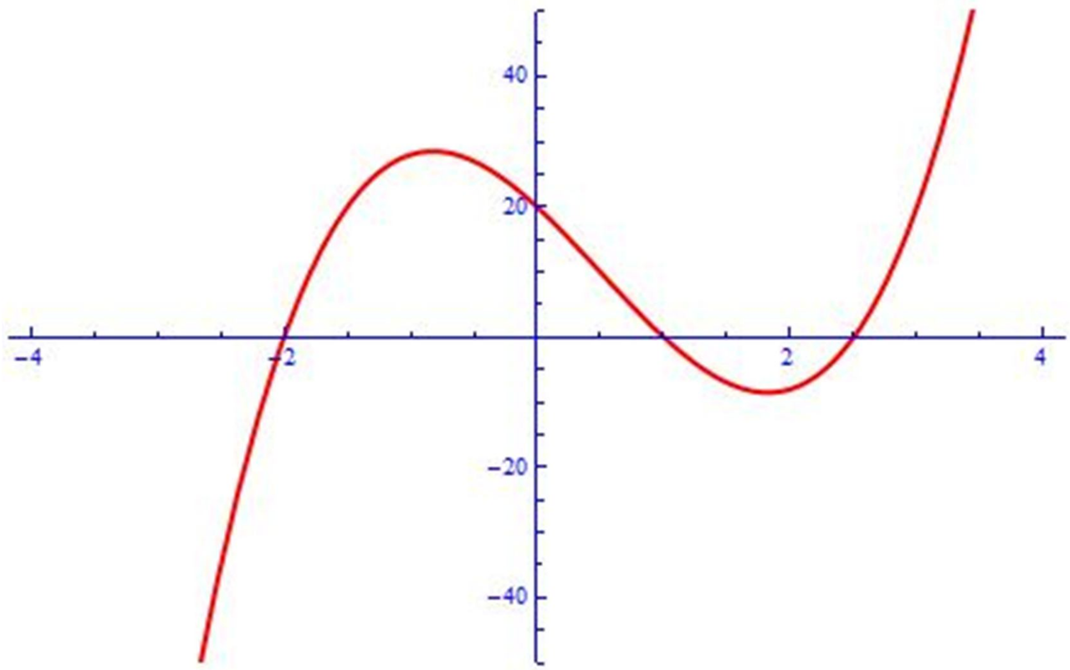
Note: **Sometimes** the Second Derivative Test **tells us nothing**!

Consider the following:

Given $f(x) = x^4$, determine and critical values, and use the Second Derivative Test to determine whether the c.v.(s) are locations of local maxima and/or minima.

Q. Well then, what is happening when $f''(x)=0$?

Consider the following Picture:



Definition 4.4.1

Given a **twice differentiable** function, $f(x)$, for any value $x=c$ where $f''(c)=0$, we call $x=c$ a **possible point of inflection**.

A **Point of Inflection** is where a function's **concavity changes** (and since functional behaviour changes, P.O.I.'s are also considered "critical"!)

Example 4.4.1

Given $f(x) = x^3 - \frac{5}{2}x^2 - 2x + 1$ determine all c.v.'s. Use the Second Derivative Test for max/min. Determine any P.P.O.I, and state intervals of concavity.

Class/Homework for Section 4.4

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