Specific Provincial Curriculum Expectations

A1: Representing Functions

- 1.1 explain the meaning of the term function, and distinguish a function from a relation that is not a function, through investigation of linear and quadratic relations using a variety of representations (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies (e.g., identifying a one-to-one or many-to-one mapping; using the verticalline test)
- 1.2 represent linear and quadratic functions using function notation, given their equations, tables of values, or graphs, and substitute into and evaluate functions [e.g., evaluate $f(\frac{1}{2})f(12)$, given $f(x)=2x^2+3x-1f(x)=2x2+3x-1$]
- 1.3 explain the meanings of the terms *domain* and *range*, through investigation using numeric, graphical, and algebraic representations of the functions $f(x)=x,\ x^2,\ \sqrt{x},\ \frac{1}{x}f(x)=x,\ x2,\ x,\ 1x$; describe the domain and range of a function appropriately (e.g., for $y=x^2+1y=x2+1$, the domain is the set of all real numbers and the range is the set $y\geq 1y\geq 1$); and explain any restrictions on the domain and range in contexts arising from real-world applications
- 1.4 relate the process of determining the inverse of a function to their understanding of reverse processes (e.g., applying inverse operations)
- 1.5 determine the numeric or graphical representation of the inverse of a linear or quadratic function, given the numeric, graphical, or algebraic representation of the function, and make connections, through investigation using a variety of tools (e.g., graphing technology, Mira, tracing paper), between the graph of a function and the graph of its inverse (e.g., the graph of the inverse is the reflection of the graph of the function in the line $y=xy=x$)

- 1.6 determine, through investigation, the relationship between the domain and range of a function and the domain and range of the inverse relation, and determine whether or not the inverse relation is a function
- 1.7 determine, using function notation when appropriate, the algebraic representation of the inverse of a linear or quadratic function, given the algebraic representation of the function and make connections, through investigation using a variety of tools (e.g., graphing technology, Mira, tracing paper), between the algebraic representations of a function and its inverse (e.g., the inverse of a linear function involves applying the inverse operations in the reverse order)
- 1.8 determine, through investigation using technology, the roles of the parameters a,k,d,ca,k,d,c in functions of the form $y=af(k(x-d))+c$y=af(k(x−d))+c, and describe these roles in terms of transformations on the graphs of $f(x)=x$, x^2, $\sqrt{x}$, $\frac{1}{x}$f(x)=x, x2, x, 1x (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)
- 1.9 sketch graphs of $y=af(k(x-d))+c$y=af(k(x−d))+c by applying one or more transformations to the graphs of $f(x)=x$, x^2, $\sqrt{x}$, $\frac{1}{x}$f(x)=x, x2, x, 1x, and state the domain and range of the transformed functions

A2: Solving Problems Involving Quadratic Functions

- 2.1 determine the number of zeros (i.e., x-intercepts) of a quadratic function, using a variety of strategies (e.g., inspecting graphs; factoring; calculating the discriminant)
- 2.2 determine the maximum or minimum value of a quadratic function whose equation is given in the form $f(x)=ax^2+bx+c$f(x)=ax2+bx+c, using an algebraic method (e.g., completing the square; factoring to determine the zeros and averaging the zeros)

- 2.3 solve problems involving quadratic functions arising from real-world applications and represented using function notation
- 2.4 determine, through investigation, the transformational relationship among the family of quadratic functions that have the same zeros, and determine the algebraic representation of a quadratic function, given the real roots of the corresponding quadratic equation and a point on the function
- 2.5 solve problems involving the intersection of a linear function and a quadratic function graphically and algebraically (e.g., determine the time when two identical cylindrical water tanks contain equal volumes of water, if one tank is being filled at a constant rate and the other is being emptied through a hole in the bottom)

A3: Determining Equivalent Algebraic Expressions

- 3.1 simplify polynomial expressions by adding, subtracting, and multiplying
- 3.2 verify, through investigation with and without technology, that $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$ $ab=a\times b$, $a,b\geq 0$ $a,b\geq 0$, and use this relationship to simplify radicals and radical expressions obtained by adding, subtracting, and multiplying
- 3.3 simplify rational expressions by adding, subtracting, multiplying, and dividing, and state the restrictions on the variable values
- 3.4 determine if two given algebraic expressions are equivalent (i.e., by simplifying; by substituting values).

B1: Representing Exponential Functions

- 1.1 graph, with and without technology, an exponential relation, given its equation in the form $y=a_x$ $y=ax$ ($a>0$, $a\neq 1$ $a>0$, $a\neq 1$), define this relation as the function $f(x)=a_x$ $f(x)=ax$, and explain why it is a function
- 1.2 determine, through investigation using a variety of tools (e.g., calculator, paper and pencil, graphing technology) and strategies

(e.g., patterning; finding values from a graph; interpreting the exponent laws), the value of a power with a rational exponent (i.e. $x^{m/n}$ where $x>0$ and m and n are integers)

- 1.3 simplify algebraic expressions containing integer and rational exponents, and evaluate numeric expressions containing integer and rational exponents and rational bases
- 1.4 determine, through investigation, and describe key properties relating to domain and range, intercepts, increasing/decreasing intervals, and asymptotes (e.g., the domain is the set of real numbers; the range is the set of positive real numbers; the function either increases or decreases throughout its domain) for exponential functions represented in a variety of ways [e.g., tables of values, mapping diagrams, graphs, equations of the form $f(x)=a^x$, function machines]

B2: Connecting Graphs and Equations of Exponential Functions

- 2.1 distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations)
- 2.2 determine, through investigation using technology, the roles of the parameters a,k,d,c in functions of the form $y=af(k(x-d))+c$, and describe these roles in terms of transformations on the graph of $f(x)=a^x$ (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)
- 2.3 sketch graphs of $y=af(k(x-d))+c$ by applying one or more transformations to the graph of $f(x)=a^x$, and state the domain and range of the transformed functions
- 2.4 determine, through investigation using technology, that the equation of a given exponential function can be expressed using different bases [e.g., $f(x)=9^x$ can be expressed

as $f(x)=3^{2x}f(x)=32x$], and explain the connections between the equivalent forms in a variety of ways (e.g., comparing graphs; using transformations; using the exponent laws)

- 2.5 represent an exponential function with an equation, given its graph or its properties

B3: Solving Problems Involving Exponential Functions

- 3.1 collect data that can be modelled as an exponential function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data
- 3.2 identify exponential functions, including those that arise from real-world applications involving growth and decay (e.g., radioactive decay, population growth, cooling rates, pressure in a leaking tire), given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range (e.g., ambient temperature limits the range for a cooling curve)
- 3.3 solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by interpreting the graphs or by substituting values for the exponent into the equations

C1: Representing Sequences

- 1.1 make connections between sequences and discrete functions, represent sequences using function notation, and distinguish between a discrete function and a continuous function [e.g., $f(x)=2xf(x)=2x$, where the domain is the set of natural numbers, is a discrete linear function and its graph is a set of equally spaced points; $f(x)=2xf(x)=2x$, where the domain is the set of real numbers, is a continuous linear function and its graph is a straight line]

- 1.2 determine and describe (e.g., in words; using flow charts) a recursive procedure for generating a sequence, given the initial terms (e.g., 1, 3, 6, 10, 15, 21, ⋯⋯), and represent sequences as discrete functions in a variety of ways (e.g., tables of values, graphs)
- 1.3 connect the formula for the nth term of a sequence to the representation in function notation, and write terms of a sequence given one of these representations or a recursion formula
- 1.4 represent a sequence algebraically using a recursion formula, function notation, or the formula for the nth term [e.g., represent 2, 4, 8, 16, 32, 64,⋯⋯ as $t_1=2; t_n=2t_{n-1}$, as $f(n)=2^n$ or as $t_n=2^n$ where n is a natural number], and describe the information that can be obtained by inspecting each representation (e.g., function notation or the formula for the nth term may show the type of function; a recursion formula shows the relationship between terms)
- 1.5 determine, through investigation, recursive patterns in the Fibonacci sequence, in related sequences, and in Pascal’s triangle, and represent the patterns in a variety of ways (e.g., tables of values, algebraic notation)
- 1.6 determine, through investigation, and describe the relationship between Pascal’s triangle and the expansion of binomials, and apply the relationship to expand binomials raised to whole-number exponents

C2: Investigating Arithmetic and Geometric Sequences and Series

- 2.1 identify sequences as arithmetic, geometric, or neither, given a numeric or algebraic representation
- 2.2 determine the formula for the general term of an arithmetic sequence [i.e., $t_n=a+(n-1)d$] or geometric sequence (i.e., $t_n=ar^{n-1}$), through investigation using a variety of tools (e.g., linking cubes, algebra tiles, diagrams, calculators) and strategies (e.g., patterning; connecting the steps in a numerical example to the steps in the algebraic development), and apply the formula to calculate any term in a sequence

- 2.3 determine the formula for the sum of an arithmetic or geometric series, through investigation using a variety of tools (e.g., linking cubes, algebra tiles, diagrams, calculators) and strategies (e.g., patterning; connecting the steps in a numerical example to the steps in the algebraic development), and apply the formula to calculate the sum of a given number of consecutive terms
- 2.4 solve problems involving arithmetic and geometric sequences and series, including those arising from real-world applications

C3: Solving Problems Involving Financial Applications

- 3.1 make and describe connections between simple interest, arithmetic sequences, and linear growth, through investigation with technology (e.g., use a spreadsheet or graphing calculator to make simple interest calculations, determine first differences in the amounts over time, and graph amount versus time)
- 3.2 make and describe connections between compound interest, geometric sequences, and exponential growth, through investigation with technology (e.g., use a spreadsheet to make compound interest calculations, determine finite differences in the amounts over time, and graph amount versus time)
- 3.3 solve problems, using a scientific calculator, that involve the calculation of the amount, AA (also referred to as future value, $FVFV$), the principal, PP (also referred to as present value, $PVPV$), or the interest rate per compounding period, ii, using the compound interest formula in the form $A=P(1+i)_n A=P(1+i)n$ [or $FV=PV(1+i)_n FV=PV(1+i)n$]
- 3.4 determine, through investigation using technology (e.g., scientific calculator, the TVM Solver on a graphing calculator, online tools), the number of compounding periods, n, using the compound interest formula in the form $A=P(1+i)_n A=P(1+i)n$ [or $FV=PV(1+i)_n FV=PV(1+i)n$]; describe strategies (e.g., guessing and checking; using the power of a power rule for exponents; using graphs) for calculating this number; and solve related problems

- 3.5 explain the meaning of the term *annuity*, and determine the relationships between ordinary simple annuities (i.e., annuities in which payments are made at the end of each period, and compounding and payment periods are the same), geometric series, and exponential growth, through investigation with technology (e.g., use a spreadsheet to determine and graph the future value of an ordinary simple annuity for varying numbers of compounding periods; investigate how the contributions of each payment to the future value of an ordinary simple annuity are related to the terms of a geometric series)
- 3.6 determine, through investigation using technology (e.g., the TVM Solver on a graphing calculator, online tools), the effects of changing the conditions (i.e., the payments, the frequency of the payments, the interest rate, the compounding period) of ordinary simple annuities (e.g., long-term savings plans, loans)
- 3.7 solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary simple annuity (e.g., calculate the total interest paid over the life of a loan, using a spreadsheet, and compare the total interest with the original principal of the loan)

D1: Determining and Applying Trigonometric Ratios

- 1.1 determine the exact values of the sine, cosine, and tangent of the special angles: 0º, 30º, 45º, 60º, and 90º
- 1.2 determine the values of the sine, cosine, and tangent of angles from 0º to 360º, through investigation using a variety of tools (e.g., dynamic geometry software, graphing tools) and strategies (e.g., applying the unit circle; examining angles related to special angles)
- 1.3 determine the measures of two angles from 0º to 360º for which the value of a given trigonometric ratio is the same
- 1.4 define the secant, cosecant, and cotangent ratios for angles in a right triangle in terms of the sides of the triangle (e.g., $\sec A = \frac{\text{hypotenuse}}{\text{adjacent}}$), and relate these ratios to the cosine, sine, and tangent ratios (e.g., $\sec A = \frac{1}{\cos A}$)

- 1.5 prove simple trigonometric identities, using the Pythagorean identity $\sin^2 x+\cos^2 x=1$; the quotient identity $\tan x=\frac{\sin x}{\cos x}$; and the reciprocal identities
- 1.6 pose problems involving right triangles and oblique triangles in two dimensional settings, and solve these and other such problems using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case)
- 1.7 pose problems involving right triangles and oblique triangles in three-dimensional settings, and solve these and other such problems using the primary trigonometric ratios, the cosine law, and the sine law

D2: Connecting Graphs of Equations and Sinusoidal Functions

- 2.1 describe key properties (e.g., cycle, amplitude, period) of periodic functions arising from real-world applications (e.g., natural gas consumption in Ontario, tides in the Bay of Fundy), given a numeric or graphical representation
- 2.2 predict, by extrapolating, the future behaviour of a relationship modelled using a numeric or graphical representation of a periodic function (e.g., predicting hours of daylight on a particular date from previous measurements; predicting natural gas consumption in Ontario from previous consumption)
- 2.3 make connections between the sine ratio and the sine function and between the cosine ratio and the cosine function by graphing the relationship between angles from 0º to 360º and the corresponding sine ratios or cosine ratios, with or without technology (e.g., by generating a table of values using a calculator; by unwrapping the unit circle), defining this relationship as the function $f(x)=\sin x$ or $f(x)=\cos x$, and explaining why the relationship is a function
- 2.4 sketch the graphs of $f(x)=\sin x$ and $f(x)=\cos x$ for angle measures expressed in degrees, and determine and describe their key properties (i.e., cycle, domain, range, intercepts, amplitude,

period, maximum and minimum values, increasing/decreasing intervals)

- 2.5 determine, through investigation using technology, the roles of the parameters a, k, d, c in functions of the form $y=af(k(x-d))+c$, where $f(x)=\sin x$ or $f(x)=\cos x$ with angles expressed in degrees, and describe these roles in terms of transformations on the graphs of $f(x)=\sin x$ and $f(x)=\cos x$ (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)
- 2.6 determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given in the form $y=a\sin(k(x-d))+c$ or $y=a\cos(k(x-d))+c$
- 2.7 sketch graphs of $y=af(k(x-d))+c$ by applying one or more transformations to the graphs of $f(x)=\sin x$ and $f(x)=\cos x$, and state the domain and range of the transformed functions
- 2.8 represent a sinusoidal function with an equation, given its graph or its properties

D3: Solving Problems Involving Trigonometric Functions

- 3.1 collect data that can be modelled as a sinusoidal function (e.g., voltage in an AC circuit, sound waves), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials, measurement tools such as motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data
- 3.2 identify periodic and sinusoidal functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range

- 3.3 determine, through investigation, how sinusoidal functions can be used to model periodic phenomena that do not involve angles
- 3.4 predict the effects on a mathematical model (i.e., graph, equation) of an application involving periodic phenomena when the conditions in the application are varied (e.g., varying the conditions, such as speed and direction, when walking in a circle in front of a motion sensor)
- 3.5 pose problems based on applications involving a sinusoidal function, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation