1.1 Functions, Domain, Range

Relation: a rule that shows how two quantities are related. Mathematically, a relation is a rule that associates each element *x* with one or more elements *y* to create a set of ordered pairs (*x, y*).

Function: a special subset (group) of relations that assigns to each element in the domain exactly one element in the range. It is a set of ordered pairs where every value of *x* has only one value for *y*.

Q1. Which of the following relationships are functions?

x	
0	2
1	3
2	4
1	6
5	7

Function? _____

x	
0	2
1	3
2	4
3	5
4	6

Function? _____

STEP 1: check for multiple x's with the same value
STEP 2: if x's have different y values than the relationship does not represent a function

NOTE: ALL functions are relations but not all relations are functions

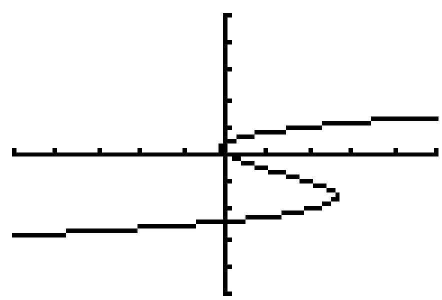
Finding that special functional relationship:

1. You can determine if a relation is a function by creating a table of values and looking for a value for *x* that will have 2 values for *y*. This can be an exhaustive task.

2. Alternatively, you can quickly sketch the relation and determine if it passes the **vertical line test.**

Vertical Line Test:

if two points on the graph of a relation lie on the same vertical line, then the relation represented by the graph is NOT a function.

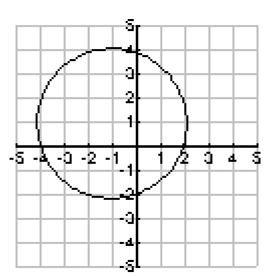


Q2. Which of the following represent a function?

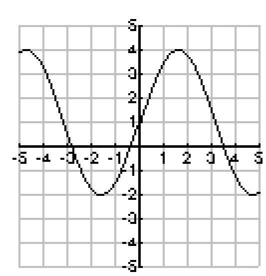
Function or Relation?

x	y
-1	-10
5	-4
0	0
-1	3
22	8

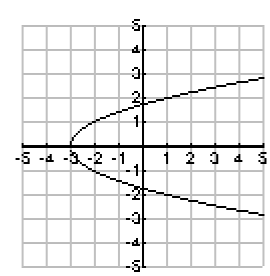
Function or Relation?



Function or Relation?

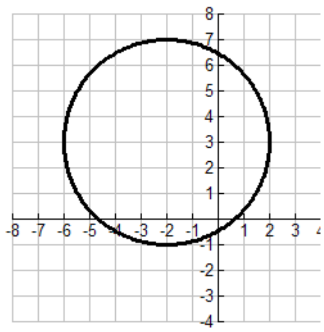


Function or Relation?



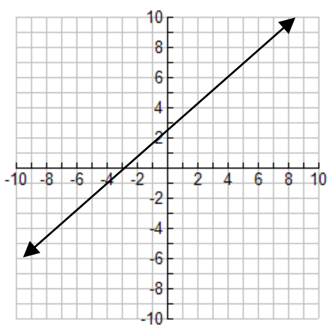
Domain and Range:

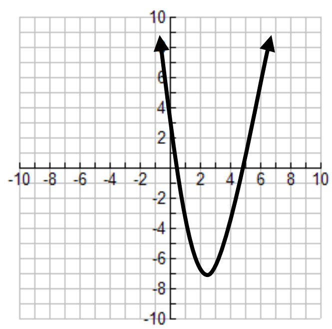
State the domain and range for each of the following.

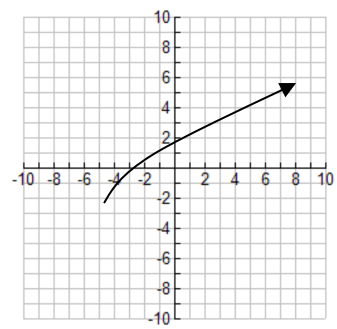


Domain: the set of the first elements in a relation/function.
Range: the set of the second elements in a relation/function.

D = { x | } R = { y | }







D =

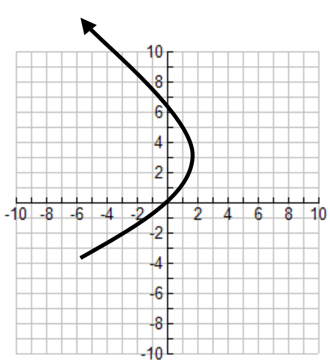
R =

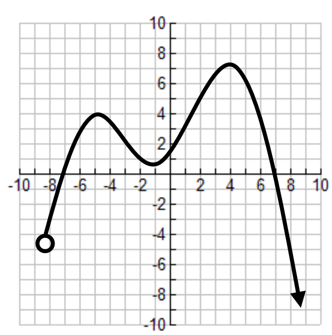
D =

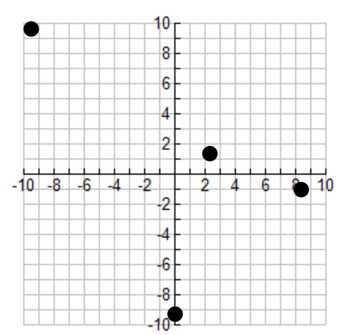
R =

D =

R =







D =

R =

D =

R =

D =

R =

Determining Domain and Range from equations:

Q3. State the domain and range for the following:

a) $y = -3x + 7$

b) $x^2 + y^2 = 100$

c) $y = \sqrt{x}$

d) $y = \dfrac{1}{x + 2}$

1.2 Functions and Function Notation

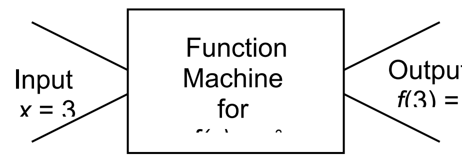
Function notation: a special relation deserves special notation

$y = x^2$ is a function. So, we can represent it using function notation.

$f(x) = x^2$ Read as: function *f* at value *x* is equal to x^2

We can think of a function as a machine.

If the input value is the independent variable and the output is the dependent variable. If we put in a specific value for the independent variable, the function machine will produce a unique value for the dependent variable. This output value will depend on the rules of operations within the function machine.



Q1. If $f(x) = 3x^2 - x + 7$, determine the value of the dependent variable:

a) $f(-2) =$

b) $f(5) =$

c) $f(\frac{1}{2}) =$

Q2. If $f(x) = x^2 + 1$ and $g(x) = 2x - 3$, determine the value of the dependent variable:

a) $f(-2) =$

b) $f(0) =$

c) $g(5) =$

d) $f(-1)+g(4)=$

e) $f(g(-1))=$

f) $g(f(-1))=$

Mapping Diagrams

Relations and Functions given as ordered pairs can be represented using mapping diagrams. This involves using directed arrows from each value in an oval representing the domain to the corresponding value or values in an oval representing the range.

Q3. a) For the set of ordered pairs listed create a mapping diagram:

(1, -5), (2, -1), (3, -5), (4, 0), (5, 0), (6, 5), (7, 8).

In a mapping diagram, a relation is not a function when an element from the domain has two or more arrows leading to different elements of the range.

b) For the set of ordered pairs listed create a mapping diagram:

(2, 1), (2, 2), (5, 3), (8, 7), (11, 5), (14, 4), (14, 6).

1.3 Maximum or Minimum of a Quadratic Function

Recall: The min or max value of a quadratic function occurs at the vertex of the parabola

How to find the vertex?

- You can **complete the square**. We complete the square to convert standard form to vertex form. Remember that $y = a(x-h)^2 + k$ the (h, k) tells you the vertex.
- You can **partial factor**…THIS IS NEW!

Q1. Determine the vertex for the following quadratics by completing the square.

a) $y = 3x^2 - 24x + 7$

b) $y = -2x^2 + 3x + 1$

Partial Factoring

Q2. Determine the vertex for the following quadratics by partial factoring.

a) $y = 4x^2 - 12x + 3$

<u>Step 1</u>: Only look at the first two terms and factor what is common

<u>Step 2</u>: Make both 'x-intercepts' equal to zero and solve for x

<u>Step 3</u>: Take the average of the two and this will give you the x value of your vertex

<u>Step 4</u>: Plug that value back in to your original question to get the y-value of the vertex

Let's do another one:

b) $y = -2x^2 + 8x - 3$

MAXIMUM / MINIMUM WORD PROBLEMS:

- Need to determine the vertex of the parabola to find optimal area, products, dimensions, etc.
  KEYWORDS: maximum, minimum, largest, smallest, optimum

STEPS TO SOLVING MAX/MIN WORD PROBLEMS:

1. Define variables using let statements or using a properly labeled diagram
2. Create equation(s) using the given information
3. Complete the square to find maximum / minimum values
4. Substitute all real solutions into let statements to determine other unknowns
5. Write a concluding statement indicating the meaning of your solution(s)

Q3. A lifeguard wants to mark off a rectangular swimming area at a beach.
He has 200m of rope to use. What is the greatest area that can be enclosed?

Q4. The student council plans to run the annual talent show to raise money for charity. Last year, tickets sold \$11 each and 400 people attended. The student council has decided to raise the ticket price for this year's event. They have determined that for every \$1 increase in price for this year's event, attendance will drop by 20 people. What ticket price should they charge to maximize revenue?

1.4 Radicals

Definitions: Radical: ____

Radicand: ____

Warm-up: Simplify each of the following

a) $\sqrt{100}$ b) $\sqrt{4} \bullet \sqrt{25}$ c) $\sqrt{144}$ d) $\sqrt{9} \bullet \sqrt{16}$

Rules for a Simplified Radical

A radical is simplified when:

1. The radicand has no perfect square factors.
2. The radicand is not a fraction.
3. A radical does not exist in the denominator of a fraction.

<u>Rules for Working with Radicals</u>

Rule 1: The root of a product can be written as the product of two roots.

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

Q1. Simplify the following radicals. Try to determine the largest perfect square factor of the radicand when breaking the radical apart.

a) $\sqrt{72}$ b) $\sqrt{60}$ c) $\sqrt{45}$

Rule 2: The root of a fraction can be written as the fraction of two roots.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Q2. Simplify the following radicals.

a) $\sqrt{\frac{121}{36}}$ b) $\sqrt{\frac{12}{25}}$ c) $\sqrt{\frac{63}{16}}$

Rule 3: If a radical exists in the denominator of a fraction, then we "**rationalize**" the denominator by multiplying the top and bottom of the fraction by the denominator. The denominator should now be integer.

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \bullet \frac{\sqrt{b}}{\sqrt{b}}$$

Q3. Simplify the following radicals.

a) $\frac{5}{\sqrt{2}}$

b) $\frac{5\sqrt{2}}{3\sqrt{7}}$

c) $\sqrt{\frac{75}{18}}$

Operations with Radicals

Adding/Subtracting Radicals: When adding / subtracting two radicals together that have the same radicand, we add/subtract their coefficients together but keep the radicand the same. Think of them as "like" radicals. You may need to simplify all radicals first before you can add/subtract.

Q4. Simplify the following.

a) $2\sqrt{5} + 3\sqrt{5}$

b) $5\sqrt{7} - \sqrt{28}$

c) $2\sqrt{5} + 3\sqrt{3} - \sqrt{45} + \sqrt{12}$

Multiplying/Dividing Radicals: When multiplying/dividing two radicals (the radicands do not need to be the same), we multiply/divide their coefficients AND multiply/divide their radicands under one radical sign. You can wait to simplify the answer until after you multiply/divide.

Q5. Simplify the following.

a) $2\sqrt{5} \bullet 3\sqrt{7}$

b) $\frac{6\sqrt{35}}{3\sqrt{7}}$

c) $2\sqrt{5} \bullet 3\sqrt{4} - \sqrt{2} \bullet \sqrt{10}$

d) $-7\sqrt{2}(6\sqrt{8}-11)$

e) $(\sqrt{3}+5)(2-\sqrt{3})$

Fractions with adding/subtracting in the numerator: If you have multiple terms in the numerator, and one term in the denominator, you can split the fraction into multiple fractions, and then simplify each piece with the above rules.

Q6. Simplify the following.

a) $\dfrac{6 \pm 4\sqrt{5}}{2}$

b) $\dfrac{16 \pm \sqrt{20}}{4}$

1.5 Solving Quadratic Equations

When solving we are NOT looking for the maximum / minimum (vertex) of a parabola.

THREE METHODS TO SOLVE QUADRATICS:

1. Factoring
2. Quadratic Formula
3. Graphing

Q1. Solve $2x^2 - 12x - 14 = 0$ using the indicated method.

a) Factoring

b) Quadratic Formula

Recall:

For a quadratic equation of the form $ax^2 + bx + c = 0$ where **a**, **b** & **c** are known **real** numbers, the quadratic formula tells us that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

How many roots (solutions) are possible? What determines the type of roots?

Q2. Determine the value of the discriminant for each of the following quadratic equations, and then state the number and type of possible solutions.

a) $2x^2 - 7x - 15 = 0$

b) $2x^2 - 5x + 4 = 0$

c) $2x^2 + 16x + 32 = 0$

Problems involving the discriminant

Q3. Determine all possible values for **k** so that give real, unreal and equal roots.

$y = x^2 + kx + 9$

SOLVING QUADRATICS WORD PROBLEMS

- Need to determine the zeros (x-intercepts) of a quadratic to find unknown dimensions, numbers, prices, etc.

STEPS TO SOLVING WORD PROBLEMS:

1. Define variables using let statements or using a properly labeled diagram
2. Create equation(s) using the given information
3. Solve by factoring or using the quadratic formula to solve for zeroes
4. Substitute all real solutions into let statements to determine other unknowns
4. Write a concluding statement indicating the meaning of your solution(s)

Q4. A triangle has an area of $24cm^2$. Its base is 2cm longer than its height. What are the dimensions of the triangle?

Q5. A rectangular swimming pool measures 6m by 10m. The pool is to be surrounded by a deck of uniform width. The total area of the deck is to be the same as the area of the pool. Find the width of the deck (round to 2 decimal places).

1.6 Determine a Quadratic Equation Given Roots

Zeros/roots can be used to find the equation of a family of quadratic functions with the same x-intercepts. A family is a function that has a common property. To determine an individual quadratic function you will also need one other point on the function.

Q1. Find the equation in factored form for a family of quadratic functions with the given x-intercepts. Give at least three members.

a) 3 and 5

b) 1 and -4

Q2. Find the equation of the quadratic function with the given zeros and containing the given point. Express your answers in standard form.

a) 2 and -3, point (0, 3)

b) double zero at x = -2, containing the point (3, 10)

Q3. The parabolic opening to a tunnel is 32 m wide measured from side to side along the ground. At the points that are 4 m from each side, the tunnel entrance is 6 m high.

a) Sketch a diagram of the given information.

b) Determine the equation of the function that models the opening to the tunnel.

c) Find the maximum height of the tunnel, to the nearest tenth of a metre.

1.7 Solve Linear-Quadratic Systems

A linear function and a quadratic function may intersect at two points (this is called a secant) and at one point (this called a tangent line) or they may never meet.

Secant: a line that intersects a curve at two points
Tangent line: a line that touches a curve at one point and has the slope of the curve at that point.

Q1. Find the points of a linear-quadratic system of equation:

$y = x^2 - 5x + 7$

$y = 2x + 1$

Q2. Determine whether a linear function intersects a quadratic function:
(looking at number of solutions)

$y = 3x \quad + 5$

$y = 3x^2 - 2x - 4$

Q3. Determine the y-intercept for a tangent line to a quadratic function:
If a line with a slope of 4 has one point of intersection with the quadratic function
$y = \frac{1}{2}x^2 + 2x - 8$ what is the y-intercept of the line? Write the equation is slope/y-intercept form.