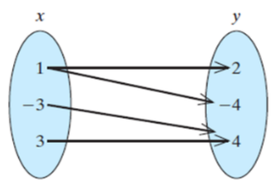
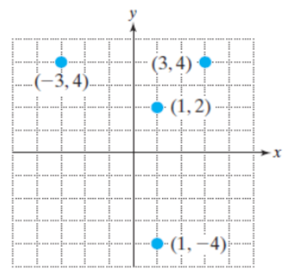
MCR3U Unit 1: Introduction to functions
Lesson 1: What is a function?

Learning Goals:
I will be able to:
- ✓ **State the characteristics of a function**
- ✓ **Distinguish between functions and relations using mappings, tables of values, graphs and equations.**
- ✓ **Use proper function notation**
- ✓ **Evaluate functions for given values**

What is a relation?

A set of ordered pairs (x, y) that are related by a certain rule are called a relation in x and y.
A relation may be defined as a set of ordered pairs, such as (1, 2). (-3,4), (1, -4), (3, 4)
A relation may also be defined as by a mapping.
e.g.,





Example 1: Analyzing a relation

The data in Table shows the length of a woman's femur and her corresponding height. Based on these data, a forensics specialist can find a linear relationship between height y, in inches, and femur length x, in centimeters:

$y = 0.906x + 24.3 \qquad 40 \leq x \leq 55$

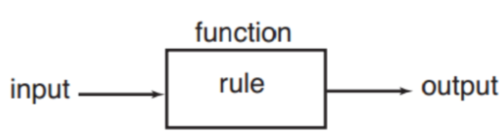
From this type of relationship, the height of a woman can be inferred based on skeletal remains.

Length of Femur (cm) x	Height (in.) y
45.5	65.5
48.2	68.0
41.8	62.2
46.0	66.0
50.4	70.0

a) Find the height of a woman femur is 46.0 cm.

b) Find the height of a woman whose femur is 51.0cm.

What is a function?

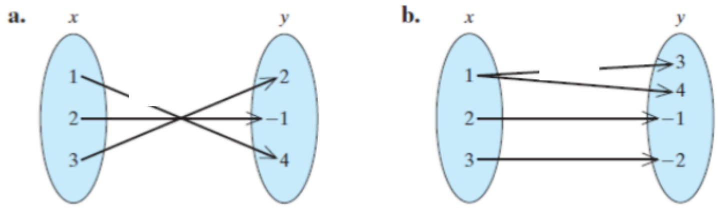


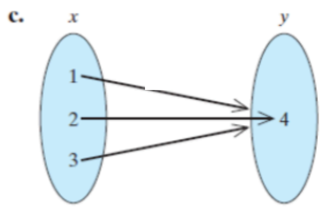
A function is a rule that operates on an input and creates an output. We can think of a function as a mathematical machine that takes input and processes it, using a given rule, and produces and output.

In order for a rule to be a function it must produce only one output for each input. All functions are relations but not all relations are functions.

Functions vs. Relations

Example 2: Determining whether a relation is a function. (i.e., determine which of the relations define y as a function of x).



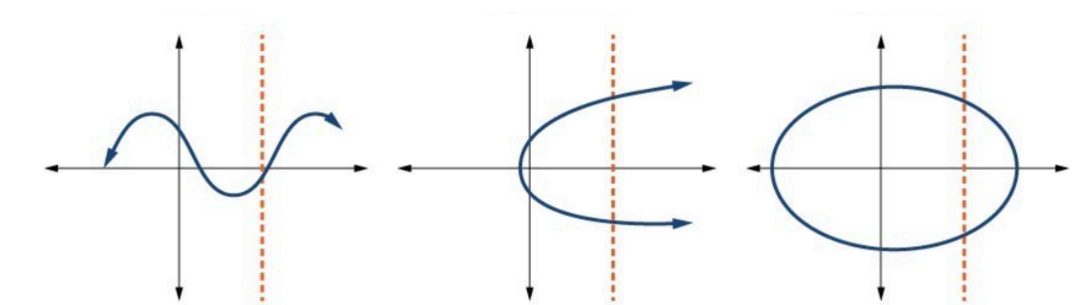


d. $\{(4, 2), (-5, 4), (0, 0), (8, 4)\}$

The vertical line test

The vertical line test is:
Consider a relation defined by a set of point (x, y) in a rectangular coordinate system. The graph defines y as a function of x **<u>if no vertical line intersects the graph in more than one point.</u>**

Example 3: Use the vertical line test to determine whether the following relations defined y as a function of x.



Function notation

Say, the function y = 2x, it takes the set of x values and doubles them to obtain the set of y values.

This can also be written in function notation, $f(x) = 2x$.

- This means the name or the rule of the function is f and the input is x.
- This is read as "f at x".
- The value of the function f, at x, is 2x.

Other example of functions:

- $f(x) = 3x^2 - 2x + 1$
- $g(x) = 3x + 4$
- $h(x) = -4.9t^2 - 4$
- $P(x) = (500 - 2x)(300 + x)$

Sometimes, under certain context or scenario, certain letter will be used, such as:

h(t) – height as a function of time
h(d) – height as a function of distance
v(t) – velocity as a function of time
d(t) – distance as a function of time
P(x) – profit as a function of items sold

Example 4: Evaluating a function
Given the function defined by $g(x) = \frac{1}{2}x - 1$, find the function values.

a) $g(0)$

b) $g(2)$

c) $g(4)$

d) $g(-2)$

Example 5: Evaluating functions
Given the functions defined by $f(x) = x^2 - 2x$ and $g(x) = 3x + 5$. Find the function values.

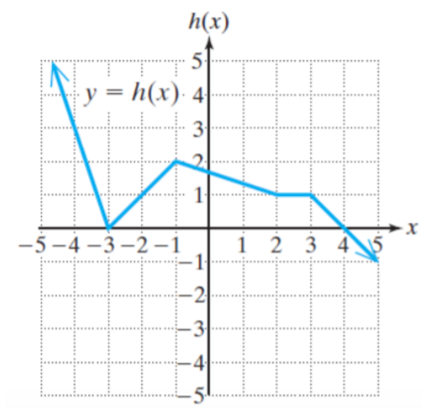
a) $f(t)$

b) $g(w + 4)$

c) $f(-t)$

Example 6: finding function values from a graph

Consider the function pictured in the Figure.



a) Find $h(-1)$.

b) Find $h(2)$.

c) For what value of x is $h(x) = 3$?

d) For what value of x is $h(x) = 0$?

Arithmetic combination of functions

Sum, difference, product, and quotient of functions
Let f and g be two functions with overlapping domains. Then, for all x common to both domains, the sum, difference, product, and quotient of f and g are defined as follows.

1. $(f+g)(x) = f(x) + g(x)$
2. $(f-g)(x) = f(x) - g(x)$
3. $(fg)(x) = f(x) \times g(x)$
4. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Composite function

Definition of composition of two functions: the composition of the function f with the function g is:

$$(f \circ g)(x) = f(g(x))$$

The composition is obtained by taking the output of g(x) and using it as input for f(x).
The notation can be read as: f circle g, f round g, f composed with g, f after g, f following g, f after g, etc.
Composing two functions is a chain process in which the output of the inner function becomes the input of the outer function.

Example 7: Given $f(x) = x + 2$ and $g(x) = 4 - x^2$, find the following.

a) $(f \circ g)(x)$

b) $(g \circ f)(x)$

c) $(g \circ f)(-2)$

Practice from Textbook: Pg. 22 # 1 – 9