Unit 3 (Chapter 3): Quadratic Functions

Lesson 3.1: Properties of quadratic functions (Ch3.1)
Determine maximum and minimum (Ch3.2)
Inverse of quadratics (Ch3.3)

Things you need to know from Grade 10:

Key Ideas

- Graphs of quadratic functions with no domain restrictions are parabolas.
- Quadratic functions have constant nonzero second differences. If the second differences are positive, the parabola opens up and the coefficient of x^2 is positive. If the second differences are negative, the parabola opens down and the coefficient of x^2 is negative.

Need to Know

- Quadratic functions can be represented by equations in function notation, by tables of values, or by graphs.
- Quadratic functions have a degree of 2.
- Quadratic functions can be expressed in different algebraic forms:
  - standard form: $f(x) = ax^2 + bx + c, a \neq 0$
  - factored form: $f(x) = a(x - r)(x - s), a \neq 0$
  - vertex form: $f(x) = a(x - h)^2 + k, a \neq 0$

Solving quadratic equation and determining roots

- ✓ Vertex form ⟶ taking square root
- ✓ Factored form ⟶ $x_1 = r$, and $x_2 = s$
- ✓ Standard form ⟶
  - Factoring
  - Completing square
  - Quadratic formula
- ✓ Discriminant: being used to determine number of roots without solving

i.e., $b^2 - 4ac \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$

Example 1: Use three different methods to find out x-intercept(s) of the following quadratic function

$$f(x) = 2x^2 + 10x + 8.$$

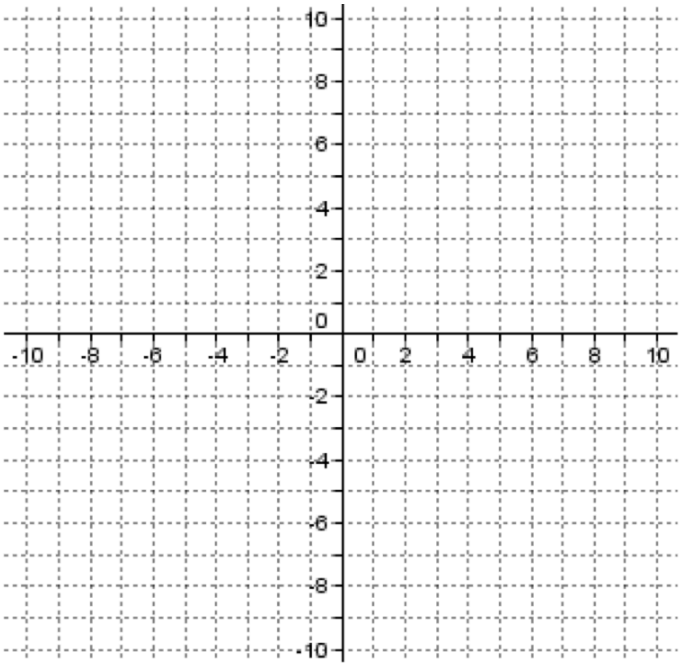
Method 1:

Method 2:

Method 3:

Sketch the function:

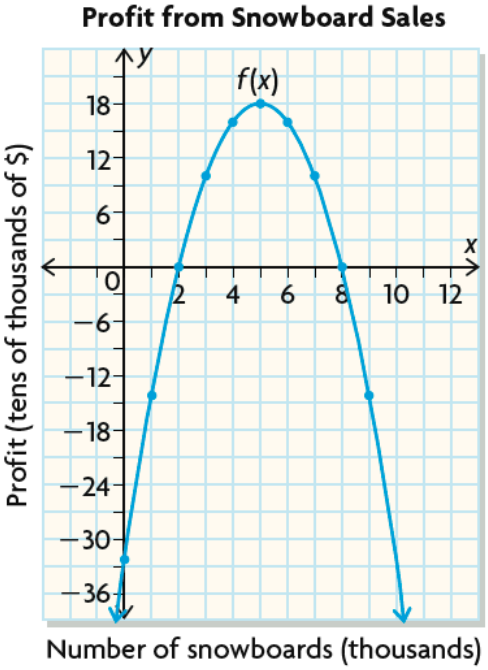
- x-intercept(s):
- y-intercept:
- Line of symmetry:
- Vertex:
- Direction of Opening:



Example 2:

Francisco owns a business that sells snowboards. His accountants have presented him with data on the business' profit in a table and a graph.

Snowboards Sold, x (1000s)	0	1	2	3	4	5	6	7	8	9
Profit, $f(x)$ ($10 000s)	−32	−14	0	10	16	18	16	10	0	−14



? What function models Francisco's profit?

Example 3: Solving a problem to determine when the maximum value occurs

The demand function for a new magazine is $p(x) = -6x + 40$, where $p(x)$ represents the selling price, in thousands of dollars, of the magazine and x is the number sold, in thousands. The cost function is $C(x) = 4x + 48$. Calculate the maximum profit and the number of magazines sold that will produce the maximum profit.

Example 4: A ticket to a school dance is $8. Usually, 300 students attend. The dance committee knows that for every $1 increase in the price of a ticket, 30 fewer students attend the dance. What ticket price will maximize the revenue?

Example 5: A high school is planning to build a new playing field surrounded by a running track. The track coach wants two laps around the track to be 1000m. The football coach wants the rectangular infield area to be as large as possible. Determine the dimensions.

Example 6:

The rate of change in the surface area of a cell culture can be modelled by the function $S(t) = -0.005(t - 6)^2 + 0.18$, where $S(t)$ is the rate of change in the surface area in square millimetres per hour at time t in hours, and $0 \leq t \leq 12$.

a) State the domain and range of $S(t)$.

b) Determine the model that describes time in terms of the surface area.

c) Determine the domain and range of the new model.