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Chapter 4: Exponential functions

Lesson 4.1: Exponent laws and simplify exponential expressions

Summary of **Exponent Law** (when bases are the **SAME**!)

- When multiplying powers with the same base, add exponents.

  $b^m \times b^n = b^{m+n}$

- When dividing powers with the same base, subtract exponents.

  $b^m \div b^n = b^{m-n}$ if $b \neq 0$

- To raise a power to a power, multiply exponents.

  $(b^m)^n = b^{mn}$

✓ An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$b^{-n} = \frac{1}{b^n}, where\ b \neq 0$$

✓ A fractional base raised to a negative exponent is equivalent to the reciprocal of the same base raise to the opposite exponent.

$$(\frac{a}{b})^{-n} = \frac{1}{(\frac{a}{b})^n} = (\frac{b}{a})^n, where\ a \neq 0, b \neq 0$$

- A number raised to a rational exponent is equivalent to a radical. The rational exponent $\frac{1}{n}$ indicates the nth root of the base. If $n > 1$ and $n \in N$, then $b^{\frac{1}{n}} = \sqrt[n]{b}$, where $b \neq 0$.
- If the numerator of a rational exponent is not 1, and if m and n are positive integers, then $b^{\frac{m}{n}} = (\sqrt[n]{b})^m = \sqrt[n]{b^m}$, where $b \neq 0$.

Example 1: Evaluate.

a) 5^{-3} **b)** $(-4)^{-2}$ **c)** -3^{-4}

Example 2: Evaluate $\frac{3^5 \times 3^{-2}}{(3^{-3})^2}$.

Example 3: Express the following in radical notation. Then evaluate.

a) $49^{-\frac{1}{2}}$ **b)** $(-8)^{\frac{1}{3}}$ **c)** $10\,000^{\frac{1}{4}}$

Example 4: Evaluate.

a) $(-27)^{\frac{4}{3}}$ **b)** $(16)^{-0.75}$

Example 5: Simplify $\dfrac{(2x^{-3}y^2)^3}{(x^3y^{-4})^2}$. and $\dfrac{(27a^{-3}b^{12})^{\frac{1}{3}}}{(16a^{-8}b^{12})^{\frac{1}{2}}}$.

More practice:

5. Simplify. Express answers with positive exponents.

a) $(3xy^4)^2(2x^2y)^3$ **c)** $\dfrac{(10x)^{-1}y^3}{15x^3y^{-3}}$ **e)** $\dfrac{p^{-5}(r^3)^2}{(p^2r)^2(p^{-1})^2}$

b) $\dfrac{(2a^3)^2}{4ab^2}$ **d)** $\dfrac{(3m^4n^2)^2}{12m^{-2}n^6}$ **f)** $\left(\dfrac{(x^3y)^{-1}(x^4y^3)}{(x^2y^{-3})^{-2}}\right)^{-1}$

6. Simplify. Express answers with positive exponents.

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a) $(x^4)^{\frac{1}{2}}(x^6)^{-\frac{1}{3}}$ **c)** $\dfrac{\sqrt{25m^{-12}}}{\sqrt{36m^{10}}}$ **e)** $\left(\dfrac{(32x^5)^{-2}}{(x^{-1})^{10}}\right)^{0.2}$

b) $\dfrac{9(c^8)^{0.5}}{(16c^{12})^{0.25}}$ **d)** $\sqrt[3]{\dfrac{(10x^3)^2}{(10x^6)^{-1}}}$ **f)** $\dfrac{\sqrt[10]{1024x^{20}}}{\sqrt[9]{512x^{27}}}$

9. Simplify. Express answers in rational form with positive exponents.

a) $(36m^4n^6)^{0.5}(81m^{12}n^8)^{0.25}$

b) $\left(\dfrac{(6x^3)^2(6y^3)}{(9xy)^6}\right)^{-\frac{1}{3}}$

c) $\left(\dfrac{\sqrt{64a^{12}}}{(a^{1.5})^{-6}}\right)^{\frac{2}{3}}$

d) $\left(\dfrac{(x^{18})^{\frac{-1}{6}}}{\sqrt[5]{243x^{10}}}\right)^{0.5}$

10. If $M = \dfrac{(16x^8y^{-4})^{\frac{1}{4}}}{32x^{-2}y^8}$, determine values for x and y so that

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a) $M = 1$ **b)** $M > 1$ **c)** $0 < M < 1$ **d)** $M < 0$

10. Simplify. Express answers with positive exponents.

a) $\dfrac{x^{0.5}y^{1.8}}{x^{0.3}y^{2.5}}$

b) $\dfrac{(mn^3)^{-\frac{1}{2}}}{m^{\frac{1}{2}}n^{-\frac{5}{2}}}$

c) $\dfrac{\sqrt{x^2y^4}}{(x^{-2}y^3)^{-1}}$

d) $\left(\dfrac{2abc^3}{(2a^2b^3c)^2}\right)^{-2}$

e) $\dfrac{\sqrt[4]{81p^8}}{\sqrt{9p^4}}$

f) $\dfrac{\sqrt[6]{(8x^6)^2}}{\sqrt[4]{625x^8}}$

11. Evaluate each expression for $a = 2$ and $b = 3$.
Express values in rational form.

a) $\left(\dfrac{b^3}{a^{\frac{5}{2}}}\right)^2\left(\dfrac{2a^4}{b^5}\right)$

b) $\sqrt{\dfrac{9b^3(ab)^2}{(a^2b^3)^3}}$