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Lesson 4.3: Applications of exponential functions

In Summary

Key Ideas

- The exponential function $f(x) = ab^x$ and its graph can be used as a model to solve problems involving exponential growth and decay. Note that
  - $f(x)$ is the final amount or number
  - a is the initial amount or number
  - for exponential growth, $b = 1 +$ growth rate; for exponential decay, $b = 1 -$ decay rate
  - x is the number of growth or decay periods

Need to Know

- For situations that can be modeled by an exponential function:
  - If the *growth rate* (as a percent) is given, then the base of the power in the equation can be obtained by *adding* the rate, as a decimal, to 1. For example, a growth rate of 8% involves multiplying repeatedly by 1.08.
  - If the *decay rate* (as a percent) is given, then the base of the power in the equation is obtained by *subtracting* the rate, as a decimal, from 1. For example, a decay rate of 8% involves multiplying repeatedly by 0.92.
  - One way to tell the difference between growth and decay is to consider whether the quantity in question (e.g., light intensity, population, dollar value) has increased or decreased.
  - The units for the growth/decay rate and for the number of growth/decay periods must be the same. For example, if light intensity decreases "per metre," then the number of decay periods in the equation is measured in metres, too.

Example 1: Population
The regional municipality of Wood Buffalo, Alberta, has experienced a large population increase in recent years due to the discovery of one of the world's largest oil deposit. Its population, 35 000 in 1996, has grown at an annual rate of approximately 8%. How long will it take for the population to double at this growth rate?

$f(t) = initial\ amount\ \times (1 + annual\ growth\ rate)^t$, where

- t is the number of years since 1996
- *f(t)* is the final population after t years

Practice 1:

A town with a population of 12 000 has been growing at an average rate of 2.5% for the last 10 years. Suppose this growth rate will be maintained in the future. The function that models the town's growth is

$$P(n) = 12(1.025)^n$$

where *P(n)* represents the population (in thousands) and *n* is the number of years from now.

a) determine the population of the town in 10 years.
b) Determine the number of years until the population doubles.
c) Use this equation to determine the number of years ago that the population was 8000. Answer to the nearest year.

Example 2: Depreciation

A new car costs $24 000. It loses 18% of its value each year after it is purchased. This is called depreciation. Determine the value of the car after 30 months.

Practice 2:

The value of a car after it is purchased depreciates according to the formula: $V(n) = 28\,000(0.875)^n$, where *V(n)* is the car's value in the n^{th} year since it was purchased.

a) What is the purchase price of the car?
b) What is the annual rate of depreciation?
c) What is the car's value at the end of 3 years?
d) What is its value at the end of 30 months?
e) How much value does the car lose in its first year?
f) How much value does it lose in its fifth year?

Example 3: Half-life

A 200g sample of radioactive element has a half-life of 138 days. This means that every 138days, the amount of this certain element left in a sample is half of the original amount. The mass of the element, in grams, that remains after t days can be modelled by

$$M(t) = 200(\frac{1}{2})^{\frac{t}{138}}$$

a) Determine the mass that remain after 5 years.
b) How long does it take for this 200g sample to decay to 110g?

Practice 3:

A species of bacteria has a population of 500 at noon. It doubles every 10 hours. The function that models the growth of the population, P, at any hour, t, is

$$\mathrm{P(t)} = 500(2^{\frac{t}{10}})$$

a) Why is the exponent $\frac{t}{10}$?
b) Why is the base 2?
c) Why is the multiplier 500?
d) Determine the population at midnight?
e) Determine the population at noon the next day?
f) Determine the time at which the population first exceeds 2000.

More practice from textbook:

9. A A student records the internal temperature of a hot sandwich that has been left to cool on a kitchen counter. The room temperature is 19 °C. An equation that models this situation is

$$T(t) = 63(0.5)^{\frac{t}{10}} + 19$$

where T is the temperature in degrees Celsius and t is the time in minutes.

a) What was the temperature of the sandwich when she began to record its temperature?

b) Determine the temperature, to the nearest degree, of the sandwich after 20 min.

c) How much time did it take for the sandwich to reach an internal temperature of 30 °C?

15. T A town has a population of 8400 in 1990. Fifteen years later, its population grew to 12 500. Determine the average annual growth rate of this town's population.

16. C A group of yeast cells grows by 75% every 3 h. At 9 a.m., there are 200 yeast cells.

a) Write an equation that models the number of cells, given the number of hours after 9 a.m.

b) Explain how each part of your equation is related to the given information.

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Lesson 4.7

13. Complete the table.

	Function	Exponential Growth or Decay?	Initial Value (y-intercept)	Growth or Decay Rate
a)	$V(t) = 100(1.08)^t$			
b)	$P(n) = 32(0.95)^n$			
c)	$A(x) = 5(3)^x$			
d)	$Q(n) = 600\left(\frac{5}{8}\right)^n$			

14. A hot cup of coffee cools according to the equation

$$T(t) = 69\left(\frac{1}{2}\right)^{\frac{t}{30}} + 21$$

where T is the temperature in degrees Celsius and t is the time in minutes.

a) Which part of the equation indicates that this is an example of exponential decay?
b) What was the initial temperature of the coffee?
c) Use your knowledge of transformations to sketch the graph of this function.
d) Determine the temperature of the coffee, to the nearest degree, after 48 min.
e) Explain how the equation would change if the coffee cooled faster.
f) Explain how the graph would change if the coffee cooled faster.

15. The value of a car after it is purchased depreciates according to the formula

$$V(n) = 28\,000(0.875)^n$$

where $V(n)$ is the car's value in the nth year since it was purchased.

a) What is the purchase price of the car?
b) What is the annual rate of depreciation?
c) What is the car's value at the end of 3 years?
d) What is its value at the end of 30 months?
e) How much value does the car lose in its first year?
f) How much value does it lose in its fifth year?

16. Write the equation that models each situation. In each case, describe each part of your equation.
a) the percent of a pond covered by water lilies if they cover one-third of a pond now and each week they increase their coverage by 10%
b) the amount remaining of the radioactive isotope U_{238} if it has a half-life of 4.5×10^9 years
c) the intensity of light if each gel used to change the colour of a spotlight reduces the intensity of the light by 4%

17. The population of a city is growing at an average rate of 3% per year. In 1990, the population was 45 000.
a) Write an equation that models the growth of the city. Explain what each part of the equation represents.
b) Use your equation to determine the population of the city in 2007.
c) Determine the year during which the population will have doubled.
d) Suppose the population took only 10 years to double. What growth rate would be required for this to have happened?