MCR3U

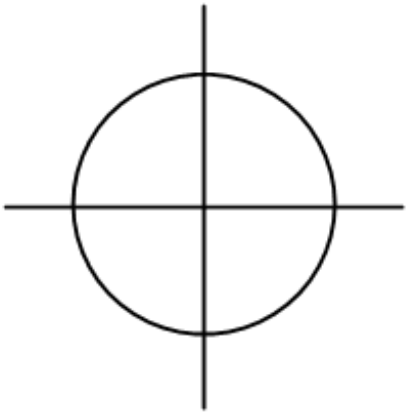
5.3 Angles between 0^o and 360^o

LEARNING GOALS

By the end of today's lesson I will be able to:

1. Use the Unit Circle and a point to find trig ratios
2. Understand that the value of sin, cos and tan changes depending on the quadrant they are in
3. Use the CAST RULE

Given a circle centred at origin (0,0) $x^2 + y^2 = r^2$

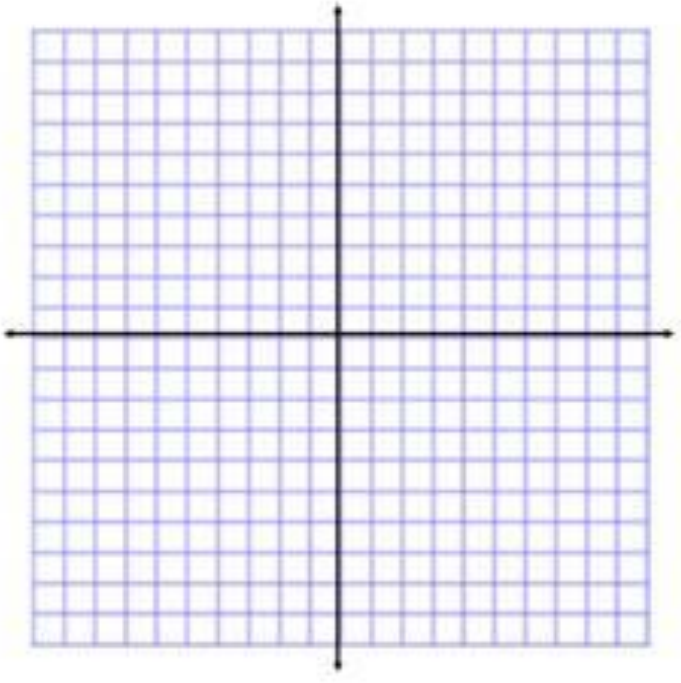


There is a point P(x,y) that can rotate from 0^o to 360^o on its circumference.

If we draw in a triangle from this point, straight into origin and then in to the x-axis we can measure the trig ratios for the given angle.

EXAMPLE 1

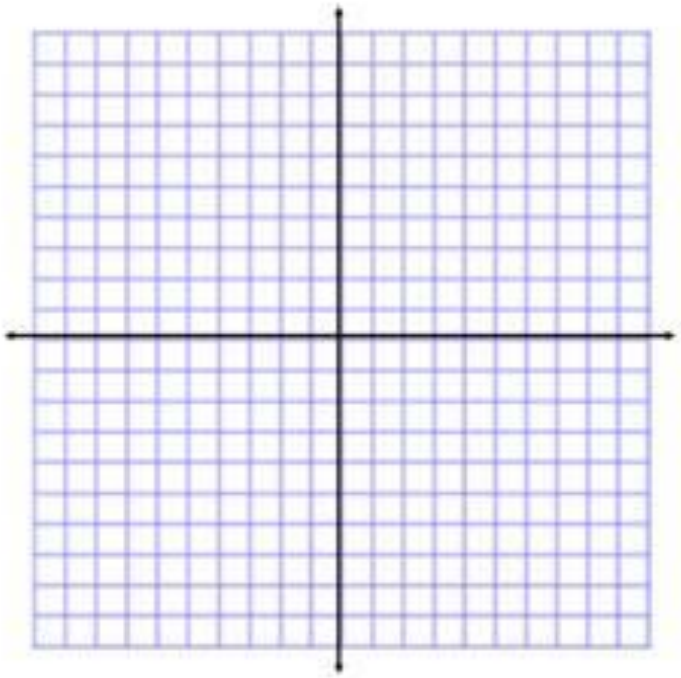
Given the circle $x^2 + y^2 = 25$ and the point P(3,4). Find all three primary trigonometric ratios and the value of the angle θ.



EXAMPLE 2

Now staying on the same circle $x^2 + y^2 = 25$ we're going to move the point into the second quadrant, and see what happens to those ratios and the angle θ.

Use the point Q(-3,4) now.



How about R(-3,-4)?

S(3,-4)?

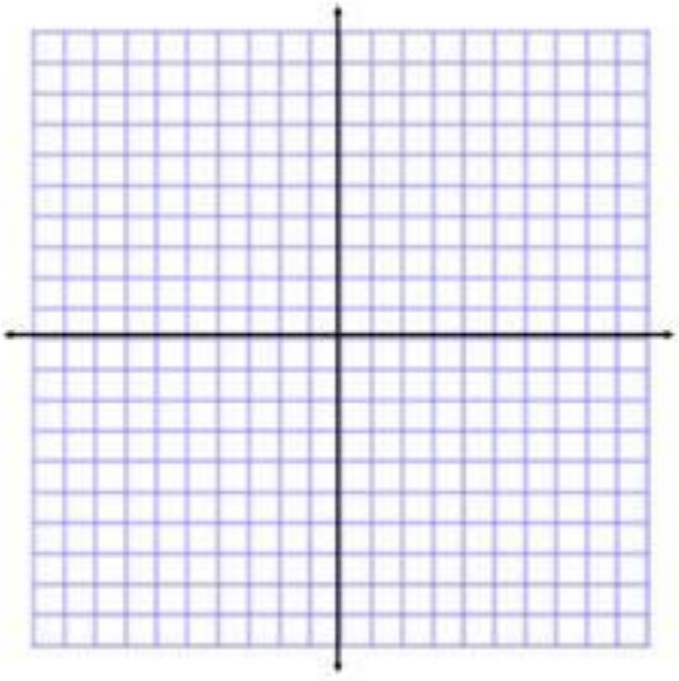
In which quadrants was $\sin(\theta)$ positive?

$\cos(\theta)$?

This is because the sine depends on the vertical value and the cosine depends on the horizontal value.

In the top two quadrants of the circle our vertical line is going in the positive direction, in the bottom two however it would be negative.

The easy way to remember this is using the CAST rule.

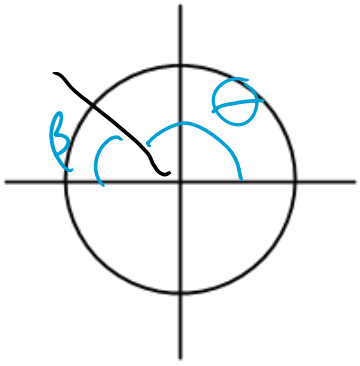


Example 3

Without using a calculator, what are the exact values of the primary trigonometry ratios for an angle of 225°?

What about finding an angle for a given ratio?

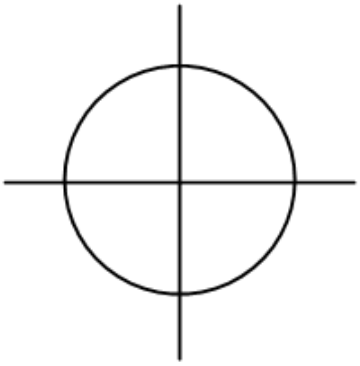
To find our angle θ, we are going to have to use what is called the **RELATED ACUTE ANGLE $\boldsymbol{\beta}$.** β is the angle the triangle makes with the x-axis.

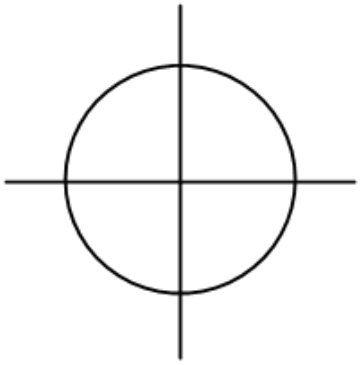


EXAMPLE 1

Find all angles given $\sin(x) = \frac{1}{2}$, $-360 \leq x \leq 360$

1) Start by drawing circles





2) Identify is this a 30/60º triangle or a 45º?

3) Cross out any restricted areas

4) Identify where $\sin(x) = \left(\frac{1}{2}\right)$

5) State the answer

EXAMPLE 2

Find all angles given $x = cos^{-1}\left(\frac{-\sqrt{2}}{2}\right)$, $-180 \leq x \leq 720$

1) Start by drawing circles

2) Identify is this a 30/60º triangle or a 45º?

3) Cross out any restricted areas

4) Identify where $\cos(x) = \frac{-\sqrt{2}}{2}$

5) State the answer

EXAMPLE 4

Find all angles given $csc(\theta) = -2$, $-180 \leq \theta \leq 180$

HW P.299 1, 2, 5, 6abc, 8aef, 9ace, 12, 13 and the questions below

14. If $0° \leq A \leq 360°$, find the possible measures of $\angle A$.

a) $\sin A = \frac{1}{2}$ b) $\cos A = \frac{1}{\sqrt{2}}$ c) $\tan A = -\sqrt{3}$ d) $\cos A = \frac{\sqrt{3}}{2}$

4. Find: $-360° \leq x \leq 360°$

a. $\sin^{-1}\left(\frac{1}{2}\right)$

b. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

c. $\sec^{-1}\left(\frac{2\sqrt{3}}{3}\right)$

d. $\cot^{-1}(.3214)$