MCR3U Unit 6 (Chapter 7): Sequence and Series
Lesson 6.1: Arithmetic and geometric sequences

Part 1: Arithmetic sequences

sequence

an ordered list of numbers

term

a number in a sequence. Subscripts are usually used to identify the positions of the terms.

arithmetic sequence

a sequence that has the same difference, the **common difference**, between any pair of consecutive terms

recursive sequence

a sequence for which one term (or more) is given and each successive term is determined from the previous term(s)

general term

a formula, labelled t_n, that expresses each term of a sequence as a function of its position. For example, if the general term is $t_n = 2n$, then to calculate the 12th term (t_{12}), substitute $n = 12$.

$$\begin{aligned} t_{12} &= 2(12) \\ &= 24 \end{aligned}$$

recursive formula

a formula relating the general term of a sequence to the previous term(s)

Key Ideas

- Every sequence is a discrete function. Since each term is identified by its position in the list (1st, 2nd, and so on), the domain is the set of natural numbers, **N** = {1, 2, 3, ...}. The range is the set of all the terms of the sequence.
  For example, $\underset{\uparrow \text{ 1st term}}{4}, \underset{\uparrow \text{ 2nd term}}{12}, 20, 28, \ldots$

  Domain: {1, 2, 3, 4, ...}
  Range: {4, 12, 20, 28, ...}

- An arithmetic sequence is a recursive sequence in which new terms are created by adding the same value (the common difference) each time.
  For example, 2, 6, 10, 14, ... is increasing with a common difference of 4,
  +4 +4 +4

  $$\begin{aligned} t_2 - t_1 &= 6 - 2 = 4 \\ t_3 - t_2 &= 10 - 6 = 4 \\ t_4 - t_3 &= 14 - 10 = 4 \\ &\vdots \end{aligned}$$

  and 9, 6, 3, 0, ... is decreasing with a common difference of −3.
  −3 −3 −3

  $$\begin{aligned} t_2 - t_1 &= 6 - 9 = -3 \\ t_3 - t_2 &= 3 - 6 = -3 \\ t_4 - t_3 &= 0 - 3 = -3 \\ &\vdots \end{aligned}$$

Need to Know

- An arithmetic sequence can be defined
  - by the general term $t_n = a + (n - 1)d$,
  - recursively by $t_1 = a$, $t_n = t_{n-1} + d$, where $n > 1$, or
  - by a discrete linear function $f(n) = dn + b$, where $b = t_0 = a - d$.

  In all cases, $n \in \mathbf{N}$, a is the first term, and d is the common difference.

Example 1: Determine whether the following recursive formula defines an arithmetic sequence

a) $t_1 = 13, t_n = 14 + t_{n-1}$

b) $t_1 = 4, t_n = t_{n-1} + n - 1$

Example 2: Determine whether each general term defines and arithmetic sequence.

$a)\ \ t_n = 8 - 2n$

b) $t_n = n^2 - 3n + 7$

Example 3: Connecting a specific term to the general term of an arithmetic sequence

What is the 33rd term of the sequence 18, 11, 4, -3, ...?

Example 4: Solving an arithmetic sequence problem

The 7th term of an arithmetic sequence is 53 and the 11th term is 97. Determine the 100th term.

Part 2: Geometric sequences

Need to Know

- A geometric sequence can be defined
  - by the general term $t_n = ar^{n-1}$,
  - recursively by $t_1 = a$, $t_n = rt_{n-1}$, where $n > 1$, or
  - by a discrete exponential function $f(n) = ar^{n-1}$.

  In all cases, $n \in \mathbf{N}$, *a* is the first term, and *r* is the common ratio.

Work on the investigation!