2015 05 15

Discrete Functions

7.1 Arithmetic Sequences

{2, 5, 8, 11, 14, ...}

t_4

Sequence - A sequence is an ordered list of numbers

Term - A term refers to a number within the sequence. Terms are usually numbered to identify their position within a sequence. ex. t_4 = the 4^{th} term

= 11

Arithmetic Sequence - An arithmetic sequence is one in which there is a common difference (d) between each successive term. $d = 3$

General Term - A formula labelled t_n, that expresses each term within the sequence as a function of its position within the sequence. $t_n =$

Recursive Formula - A recursive formula is an expression that models one term of a sequence in terms of the previous term.

$t_1 = 2$

$t_n = t_{n-1} + 3$

$t_2 = t_1 + 3 = 2 + 3 = 5$

$t_3 = t_2 + 3 = 5 + 3 = 8$

Ex. The following are examples of sequences
a) identify whether they are arithmetic or not

b) Identify t_4 for each sequence

3, 8, 12, 15, 20, 19, ...

No common difference.

$t_4 = 15$

2, 6, 10, 14, 18, 22, ...

Yes — common difference = 4

$t_4 = 14$

The General Term

The general term of an arithmetic sequence can always be written as $t_n = a + (n - 1)d$,
where t_n is the n^{th} term of the sequence
'a' is the first term and 'd' is the common difference between each term

Ex. Identify the general term of the following sequences, then use it to determine the 17^{th} term of each sequence

3, 12, 21, 30,....

$t_n = a + (n-1)d$
$t_n = 3 + (n-1)9$
$t_n = 3 + 9n - 9$
$t_n = 9n - 6$ $n \geq 1$

$t_{17} = 9(17) - 6$
$t_{17} = 147$

15, 9, 3, -3, ...

$15 + (n-1)(-6)$
$t_n = 15 - (n-1)6$
$= 15 - 6n + 6$
$= 21 - 6n$

$t_{17} = 21 - 6(17)$
$= -81$

Ex. The 7^{th} term of an arithmetic sequence is 53 and the 11^{th} term is 97. Determine:

(i) the general term

(ii) the 100^{th} term.

$t_7 = 53$

$t_{11} = 97$

$t_n = a + (n-1)d$

$53 = a + (7-1)d$

$53 = a + 6d$

$97 = a + (11-1)d$

$97 = a + 10d$

$53 = a + 6d$

$44 = 4d$

$11 = d$

$53 = a + 6(11)$

$53 = a + 66$

$53 - 66 = a$

$a = -13$

$\therefore t_n = -13 + (n-1)(11)$

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4 terms: $t_7 = 53$, $t_{11} = 97$

$a, a+d, a+2d, \ldots, 53, \ldots, \ldots, \ldots, 97$ (+d +d +d +d)

53 is t_7, 97 is t_{11}

$97 - 53 = 4d$

$44 = 4d$

$11 = d$

$t_n = -13 + (n-1)(11)$

$t_{100} = -13 + (99)(11)$

$= 1076$

Ex. An ampitheater to be built at Pinafore is designed so that there are 15 seats in the front row 18 seats in the second row, 21 seats in the third row and so on.

a) How many seats are in the 8th row?

$t_n = 15 + (n-1)(3)$

$t_8 = 15 + (7)(3)$

$= 36$

b) If the last row has 48 seats, how many rows of seating are there?

$48 = 15 + (n-1)(3)$

$\frac{33}{3} = \frac{(n-1)(3)}{3}$

$11 = n - 1$

$\therefore n = 12$

c) How many total seats are there?

15, 18, 21, ..., 42, 45, 48

+3 +3 +3

12 rows $\therefore$ 6 pairs

$\times 63$

378

Pg. 424 #1-4, 5,6,8,9 ad,10,11,14,17