MCR3U Unit 6 (Chapter 7): Sequence and Series
Lesson 6.3: Arithmetic and geometric series (7.5 – 7.6)

Formulas:

Arithmetic series

- The sum of the first n terms of an arithmetic sequence can be calculated using
  - $S_n = \frac{n[2a + (n - 1)d]}{2}$ or
  - $S_n = \frac{n(t_1 + t_n)}{2}$.

  In both cases, $n \in \mathbf{N}$, a is the first term, and d is the common difference.

Geometric series

- The sum of the first n terms of a geometric sequence can be calculated using
  - $S_n = \frac{a(r^n - 1)}{r - 1}$, where $r \neq 1$ or
  - $S_n = \frac{t_{n+1} - t_1}{r - 1}$, where $r \neq 1$.

  In both cases, $n \in \mathbf{N}$, a is the first term, and r is the common ratio.

Example 1: Solving a problem by using an arithmetic series

In an amphitheatre, seats are arranged in 50 semicircular rows facing a domed stage. The first row contains 23 seats, and each row contains 4 more seats than the previous row. How many seats are in the amphitheatre?

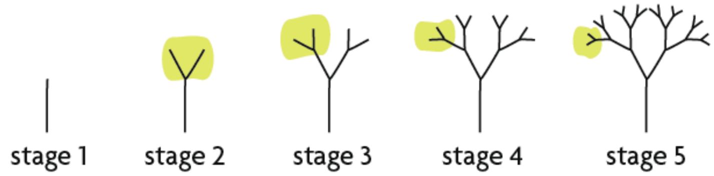
Example 2: Selecting a strategy to calculate the sum of a series when the number of terms is unknown

Arithmetic
Determine the sum of $-31 - 35 - 39 - \ldots -403$.

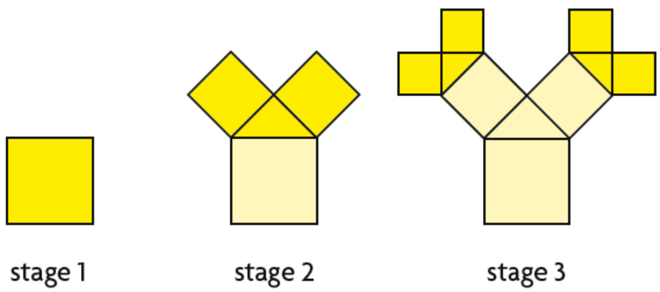
Geometric
$7\,971\,615 + 5\,314\,410 + 3\,542\,940 + \ldots + 92\,160$.

Practice from Textbook:
Pg459 – 460

9. A simple fractal tree grows in stages. At each new stage, two new line segments branch out from each segment at the top of the tree. The first five stages are shown. How many line segments need to be drawn to create stage 20?



10. A Pythagorean fractal tree starts at stage 1 with a square of side length 1 m. At every consecutive stage, an isosceles right triangle and two squares are attached to the last square(s) drawn. The first three stages are shown. Calculate the area of the tree at the 10th stage.



15. In a geometric series, $t_1 = 12$ and $S_3 = 372$. What is the greatest possible value for t_5? Justify your answer.

16. In a geometric series, $t_1 = 23$, $t_3 = 92$, and the sum of all of the terms of the series is 62 813. How many terms are in the series?

17. Factor $x^{15} - 1$.

18. Suppose you want to calculate the sum of the *infinite* geometric series

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots.$$

a) The diagram shown illustrates the first term of this series. Represent the next three terms on the diagram.

b) How can the formula for the sum of a geometric series be used in this case?

c) Does it make sense to talk about adding together an infinite number of terms? Justify your reasoning.