Simple and Compound Interest

Textbook:	**section 8.1 p. 476 – 481**	**Homework:**	**p. 481 – 482 #3, 4, 5 adef, 7**
	section 8.2 p. 483 – 489		**p. 490 – 492 #4, 11, 16, 17**

Interest

- the cost of using money
  - for a loan, it's your cost of using the bank's money
  - for a savings/investment account, it's what you earn for giving the bank the use of your money

Simple Interest

- the interest earned or paid is on the original amount of money only
- $I = Prt$
  - where
    - I is the amount of interest earned/paid
    - P is the principal (amount of money) invested/borrowed **now** to get a specific future value (FV) in a certain amount of time
      - also called the present value (PV)
    - r is the annual interest rate (as a decimal)
    - t is the time in years
- $$\begin{aligned} A &= P + I \\ &= P + \Pr t \\ &= P(1 + rt) \end{aligned}$$
  $$FV = PV(1 + rt)$$
  - where
    - A is the final amount of an investment/loan after a certain length of time
      - balance in the savings account or total amount of loan repaid
      - includes the principal and all of the accumulated interest
      - also called the future value (FV)

Year	Total Amount
	P
1	$P + I$
2	$P + I + I = P + 2I$
3	$P + I + I + I = P + 3I$

  $\therefore$ Arithmetic Sequence

Note: "/a" means **per annum** — amount for one year

Example 1: Determine the amount of simple interest earned.

a) $1 000 at 5%/a for 3 years

$$\begin{aligned} I &= Prt \\ &= (1000)(0.05)(3) \\ &= 150 \end{aligned}$$

$\therefore$ The interest earned is $150.

b) $500 at 3.75%/a for 40 months

$$\begin{aligned} I &= Prt \\ &= (500)(0.0375)\left(\frac{40}{12}\right) \\ &= 62.50 \end{aligned}$$

$\therefore$ The interest earned is $62.50.

Example 2: Sam borrowed $1 000 for 85 days by taking a cash advance on his credit card. The interest rate is 26%/a simple interest. How much will he need to pay back at the end of the loan period?

$$\begin{aligned} I &= Prt \\ &= (1000)(0.26)\left(\frac{85}{365}\right) \\ &\doteq 60.55 \end{aligned}$$

$\therefore$ Sam will have to pay back approximately $1 060.55.

Example 3: Tanya invested $1 800 at 6.5% simple interest. How long must she keep her investment in the bank before earning $100 in interest?

$$\begin{aligned} I &= Prt \\ 100 &= (1800)(0.065)t \\ t &= \frac{100}{1800(0.065)} \\ &\doteq 0.8547 \end{aligned}$$

$$\begin{aligned} \text{number of days} &= 365t \\ &\doteq (365)(0.8547) \\ &\doteq 312 \end{aligned}$$

$\therefore$ Tanya will have to keep her money invested for approximately 312 days to earn $100 in interest.

Compound Interest

- the interest earned at regular intervals is added to the principal, then, interest is earned on the growing amount
- $A = P(1+i)^n$
  $FV = PV(1+i)^n$
  - where
    - A is the future value (FV) of the investment/loan after a certain length of time
      - balance in the savings account or total amount of loan repaid
      - includes the principal and all of the accumulated interest
      - also called the future value (FV)
    - P is the principal (amount of money) invested/borrowed **now** to get a specific future value (FV) in a certain amount of time
      - also called the present value (PV)
      - $$A = P(1+i)^n$$
        $$FV = PV(1+i)^n$$
        $$PV = \frac{FV}{(1+i)^n}$$
        $$PV = FV(1+i)^{-n}$$
    - i is the interest rate per compounding period (as a decimal)
      - interest rate is **always** given per annum (for one year)
        - need to convert the annual interest rate to an equivalent amount per compounding period
    - n is the number of compounding periods
      - the period for which interest is calculated
      - n = (number of years)*(number of times interest is calculated in a year)
- $$\begin{aligned} I &= A - P \\ &= P(1+i)^n - P \\ &= P\left[(1+i)^n - 1\right] \end{aligned}$$
  - where
    - I is the total amount of accumulated interest earned/paid
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Year	Total Amount
	P
1	$P(1+i)$
2	$P(1+i)(1+i) = P(1+i)^2$
3	$P(1+i)(1+i)(1+i) = P(1+i)^3$

$\therefore$ Geometric Sequence

Most common time periods:

- annually = once a year
- semi-annually = twice a year (every six months)
- quarterly = four times a year (every three months)
- monthly = 12 times a year (every month)
- bi-weekly = 26 times a year (every two weeks)
- weekly = 52 times a year (every week)
- daily = 365 times a year (every day)

Example 4: Determine the future value if $1 000 is invested at 5%/a compounded semi-annually for three years.

$$\begin{aligned} A &= P(1+i)^n \\ &= 1000\left(1+\frac{0.05}{2}\right)^{2(3)} \\ &= 1000(1.025)^6 \\ &\doteq 1159.69 \end{aligned}$$

$\therefore$ The future value is approximately $1 159.69.

Example 5: Determine the amount of interest paid if $8 000 is borrowed for four years at 18%/a compounded monthly.

$$\begin{aligned} A &= P(1+i)^n \\ &= 8000\left(1+\frac{0.18}{12}\right)^{4(12)} \\ &= 8000(1.015)^{48} \\ &\doteq 16347.83 \end{aligned}$$

$$\begin{aligned} I &= A-P \\ &\doteq 16347.83-8000 \\ &= 8347.83 \end{aligned}$$

$\therefore$ The amount of interest paid is approximately $8 347.83.

Example 6: Jaime's parents would like to put some money away so that he will have $15 000 for university in 10 years. How much will they need to invest in an account earning 6%/a compounded annually to achieve this goal?

$$\begin{aligned} PV &= \frac{FV}{(1+i)^n} \\ &= \frac{15000}{(1+0.06)^{10}} \\ &\doteq 8375.92 \end{aligned}$$

$\therefore$ They will need to invest approximately $8 375.92.

Example 7: Billy is investing $5 000 that he would like to grow to at least $50 000 by the time he retires in 40 years. What interest rate compounded annually will provide this ?

$$FV = PV(1+i)^n$$
$$50000 = 5000(1+i)^{40}$$
$$10 = (1+i)^{40}$$
$$1+i = \sqrt[40]{10}$$
$$i = \sqrt[40]{10} - 1$$
$$i \doteq 0.0593$$

∴ Billy will need to get an interest rate of approximately 5.93% in order to achieve his goal.