7.7 Pascal's Triangle and The Binomial Expansion

Problem: Expand the binomial $(x + y)^6$

$$(x+y)^6 = (x+y)(x+y)(x+y)(x+y)(x+y)(x+y)$$

Solution:

Very long and cumbersome !

Today's Goal: To discover patterns with the coefficients and exponents to expand this more quickly than expanding with the Distributive Property over and over.

Start by expanding the following expressions using the Distributive Property (i.e., FOIL).

Remember: Anything to the power of 0 = ?

$(x + y)^0 =$

$(x + y)^1 =$

$(x + y)^2 =$

$(x + y)^3 =$

$(x + y)^4 =$

Make a list of the **coefficients** for each of the terms, in each expansion.
What do you notice about the **exponents** on the x and on the y for each term?
Also note the **number of terms** in each expansion and how it relates to the exponent.

See p. 463 to check your solution !

$(x + y)^0 =$

$(x + y)^1 =$

$(x + y)^2 =$

$(x + y)^3 =$

$(x + y)^4 =$

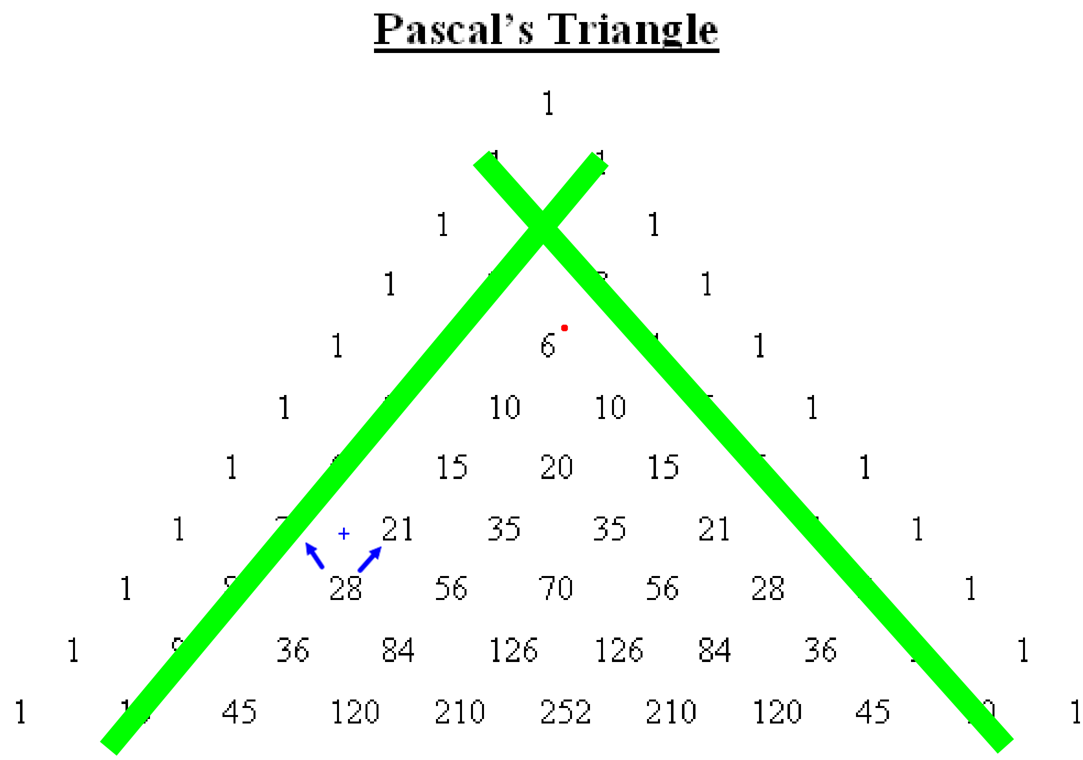
$x + y)^5 =$

$(x + y)^6 =$

A binomial is an expression that contains two terms separated by an addition or subtraction.

When a binomial is raised to an exponent, the resulting coefficients, exponents and number of terms follow a predictable pattern.

The coefficients of a binomial expansion follow the pattern seen in what is called "Pascal's Triangle"



Patterns in Pascal's Triangle

- each row begins and ends with the number 1
- the diagonal beside that are the numbers 1, 2, 3, etc.
- any number of the triangle can be determined by adding up the 2 directly above it

Use Pascal's Triangle and the patterns with the exponents to expand and simplify the following expressions:

$(x+3)^3 =$

$(a+b)^5$

$(b+a)^5$

$(5x+2y)^3$

Pg. 466 #1,<u>2-3ac</u>,<u>4-5bf</u>,8,10,11

7. A ball is dropped from a height of 3 m and bounces on the ground. At the top of each bounce, the ball reaches 60% of its previous height. Calculate the total distance travelled by the ball when it hits the ground for the fifth time.

$3 + 1.8 + 1.08$

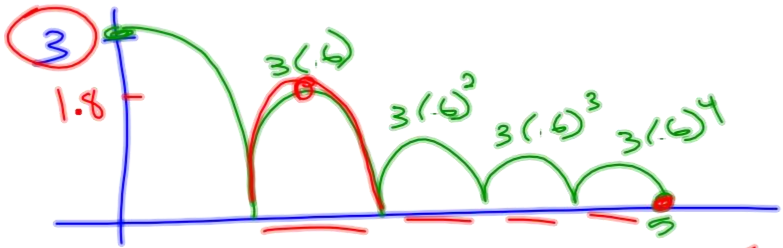
$r = 0.6$

$a = 3$

$$S_5 = \frac{3(0.6^5 - 1)}{0.6 - 1}$$

$$= 6.9$$

(2)

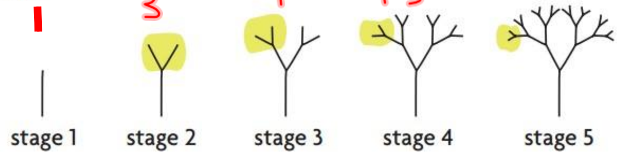


$1.8 + 1.08 +$

$$S_4 = \left(\frac{1.8(0.6^4 - 1)}{0.6 - 1}\right) \times 2 + 3$$

9 A A simple fractal tree grows in stages. At each new stage, two new line segments branch out from each segment at the top of the tree. The first five stages are shown. How many line segments need to be drawn to create stage 20?

1 3 7 15



$1 + 2 + 4 + 8$

$$S_{20} = \frac{1(2^{20} - 1)}{2 - 1}$$

12. John wants to calculate the sum of a geometric series with 10 terms, where the 10th term is 5 859 375 and the common ratio is $\frac{5}{3}$. John solved the problem by considering another geometric series with common ratio $\frac{3}{5}$. Use John's method to calculate the sum. Justify your reasoning.

$$5859375 = ar^{10-1}$$

$$5859375 = a\boxed{\left(\frac{5}{3}\right)^9}$$

$$a = 5859375\left(\frac{3}{5}\right)^9$$

$$a = 59049$$

$$S_{10} = \frac{a\left(\left(\frac{5}{3}\right)^{10} - 1\right)}{\frac{5}{3} - 1}$$

$$= 59049\left[\frac{\left(\frac{5}{3}\right)^{10} - 1}{2/3}\right]$$