4.3 Asymptotes

We will consider **three kinds** of asymptotes: **Vertical, Horizontal and Oblique**

But first, recall that **Rational Functions** have the form

$$R(x)=\frac{P(x)}{Q(x)},\quad Q(x)\neq 0\text{, where both } P(x) \text{ and } Q(x) \text{ are polynomial functions.}$$

We need this recall since **rational functions may give us asymptotes**, which polynomial functions will not.

Vertical Asymptotes

A rational function $R(x)=\frac{P(x)}{Q(x)},\ \ Q(x)\neq 0$

may have a vertical asymptote at values for x where $Q(x)=0$.

Q. When would a rational function **NOT** have a V.A. when the denominator is zero?

Definition 4.3.1

Given a rational function, $f(x)$, whenever

Example 4.3.1

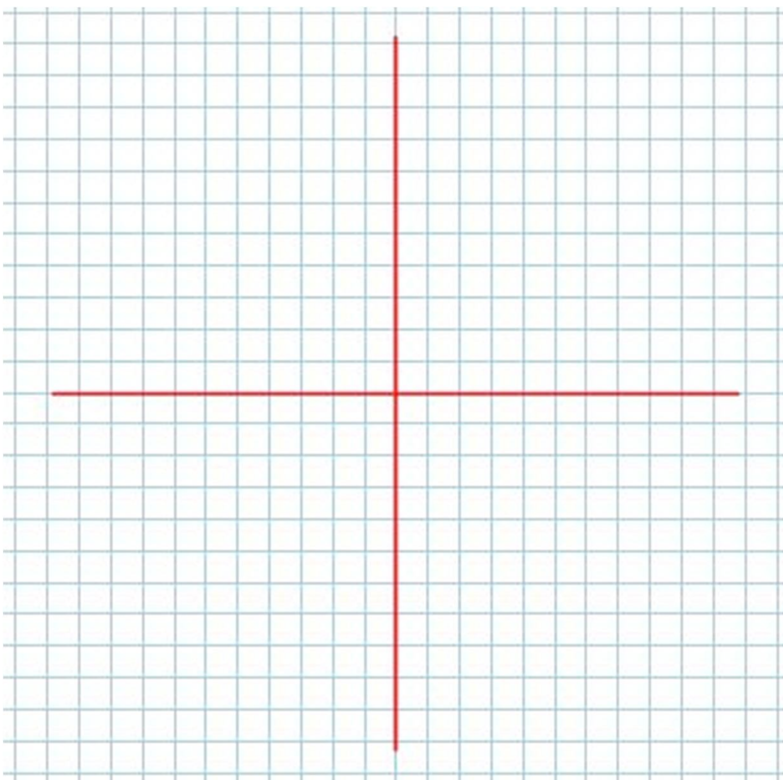
Show $x=3$ is not a vertical asymptote of $f(x)=\frac{x-3}{x^2-5x+6}$, whereas $x=2$ is a V.A.

Horizontal Asymptotes

For **horizontal asymptotes** (H.A.'s) we are concerned with the so-called **end behaviour** of a function. That is we are wondering about the functional behaviour as

Algebraically we want to understand what is happening for $\lim_{x\to\infty}\left(f(x)\right)$ and $\lim_{x\to-\infty}\left(f(x)\right)$

Picture



Note: There are **2 ways** to **approach a H.A.**! (And I don't mean the two sides of infinity!!)

Example 4.3.2

Determine the H.A. (if it exists) and the functional behaviour “near” the H.A. for

$$f(x) = \frac{1}{x-2}$$

Example 4.3.3

Determine the H.A. and discuss the functional end behaviour for $g(t) = \dfrac{3t^2 - 5t}{4t^2 + 1}$

Notes about **Limits at Infinity**

1) $\lim_{x\to\pm\infty}\left(\frac{\text{lower degree}}{\text{higher degree}}\right)=0$ e.g. $\lim_{x\to\infty}\left(\frac{3x^2-5x+1}{5x^3+2x^2-7}\right)=0$

2) $\lim_{x\to\infty}\left(\frac{\text{degree } n}{\text{degree } n}\right)=$ a non-zero number e.g. $\lim_{t\to\infty}\left(\frac{3t^3-5t+1}{6t-9t^3}\right)=$

3) $\lim_{x\to\infty}\left(\frac{\text{higher degree}}{\text{lower degree}}\right)=\infty$ (i.e. no H.A.)

BUT

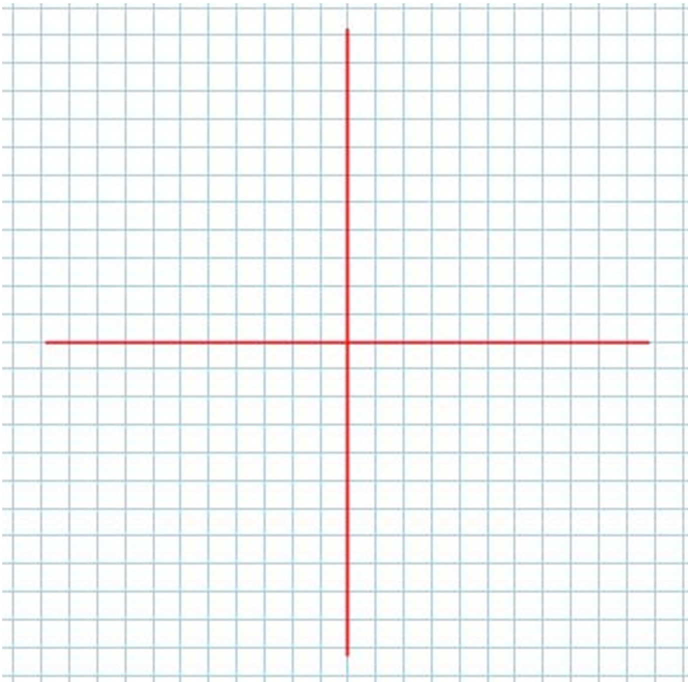
whenever we have $f(x)=\left(\frac{\text{degree }(n+1)}{\text{degree }(n)}\right)$, then we have

and we find them using **Polynomial Long Division** (Hooray!!!!!)

Example 4.3.4

Determine any asymptotes for $f(x)=\frac{3x^2-8x-7}{x-4}$.

Picture



Class/Homework for Section 4.3

(read Need to Know on pg. 192)

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