

1.2 Properties of Functions

Learning Goal: We are learning to compare and contrast the properties and characteristics of various types of functions.

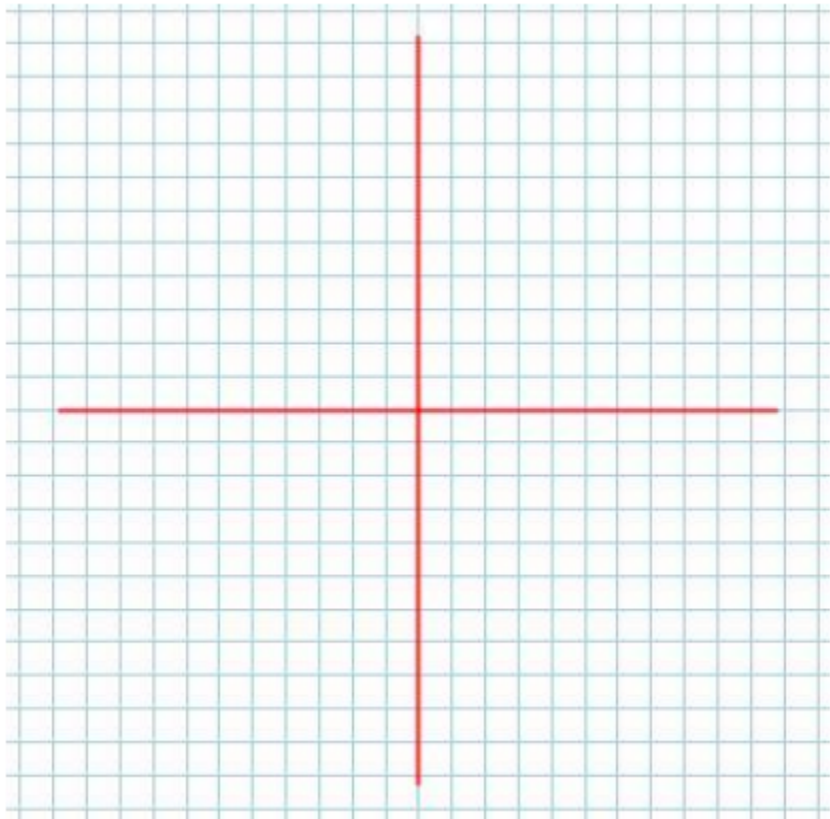
Recall that we define the graph of a function to be the SET of Ordered Pairs:

We can visualize the graph of a function by plotting its ordered pairs on the Cartesian axes.

Example 1.2.1

e.g. $f(x) = x^2$ has the graph

and looks like



Characteristics of a Function's Graph

Over the course we will be studying Polynomial, Rational, Trigonometric, Exponential and Logarithmic Functions. For now we are focused on Polynomial and Rational Functions, but for each type of function we will try and understand various functional (final) behaviours (or characteristics).

The characteristics (behaviours) we are primarily interested in studying are:

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.

*Note: Generally a geometric point of view will just mean that we'll look at pictures, but Geometry is actually **much** deeper than that!*

Intervals of Increase and Decrease

We will examine (when possible) functional behaviour from both algebraic and geometric points of view.

Definition 1.2.1

A function $f(x)$ is said to be increasing on the open interval (a, b) when

A function $f(x)$ is said to be decreasing on the open interval (a, b) when

Note the difference between **open** and **closed** intervals:

An **open** interval

A **closed** interval

Example 1.2.2

Consider the function $f(x)$, represented graphically:

Determine where $f(x)$ is increasing and decreasing.

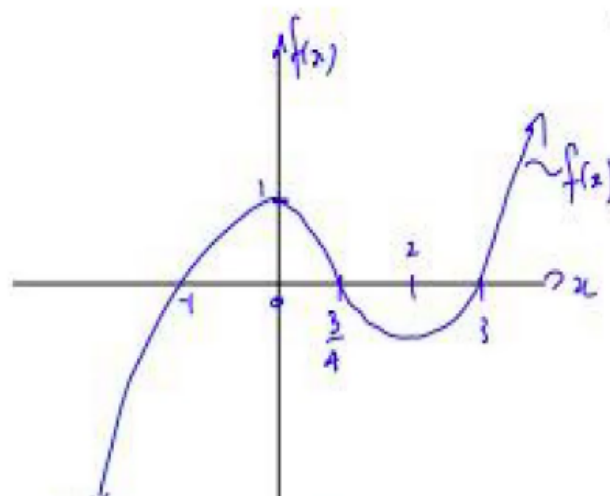


Figure 1.2.2

Odd vs. Even Functions

Note: This functional behaviour deals with SYMMETRY rather than the “power(s)” that you might see in various terms of the function.

Basic Definition:

- Even Functions are symmetric around the
- Odd Functions are symmetric around the

Graphical point of view:

<i>Even Functions</i>	<i>Odd Functions</i>

Algebraically we will consider definitions for Even and Odd Functions:

Definition 1.2.2

A function $f(x)$ is **even** if

A function $f(x)$ is **odd** if

Example 1.2.3

a) Show $f(x) = 3x^4 + 2x^2 + 5$ is even.

b) Show $g(x) = 5x^3 - 2x$ is odd.

- c) Are i) $f(t) = 5t^3 - 2t + 1$ and
ii) $h(x) = \frac{3x^3 - 2x}{x^2 - 1}$ odd or even?

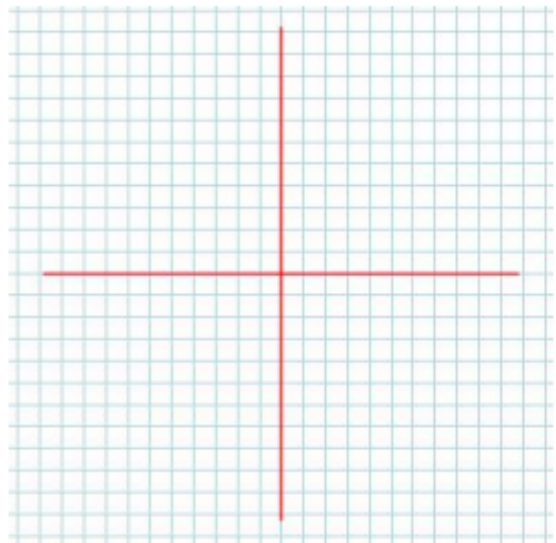
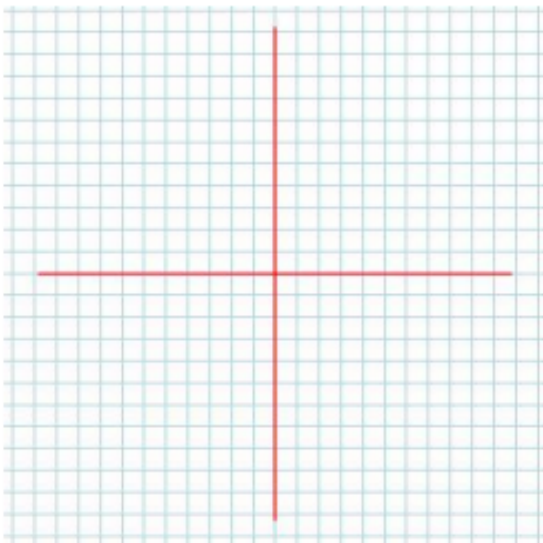
Continuity

For the time being we will consider a (quite) rough definition of what it means for a function to be continuous. In fact, we will see that understanding what it means for a function to be discontinuous may be more helpful for now. In the course *Calculus and Vectors*, a formal, algebraic definition of continuity will be considered.

Rough Definition

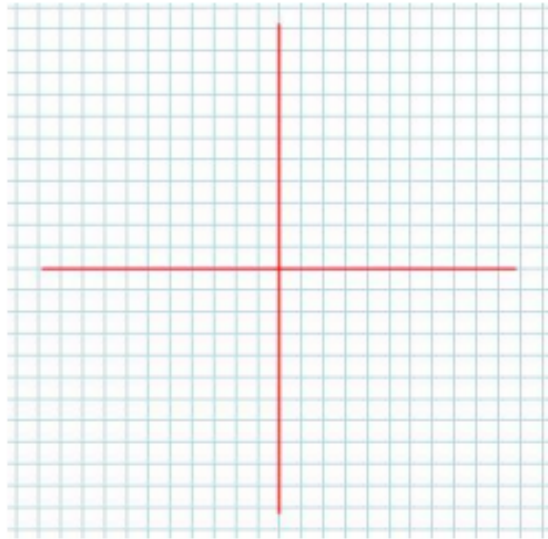
A function $f(x)$ is **continuous** (cts) on its domain D_f if

Pictures

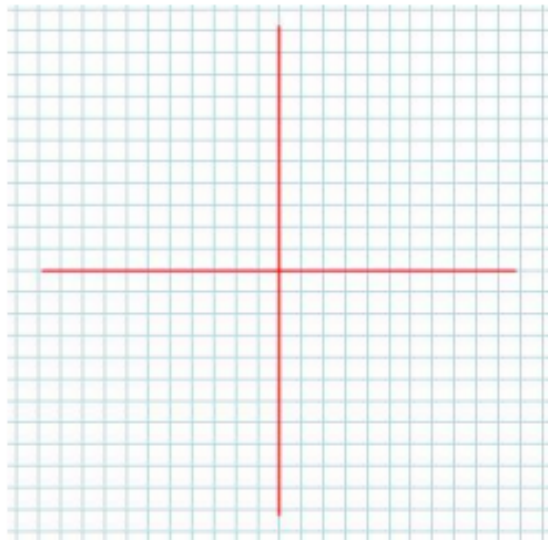


There are 3 types of **discontinuities**:

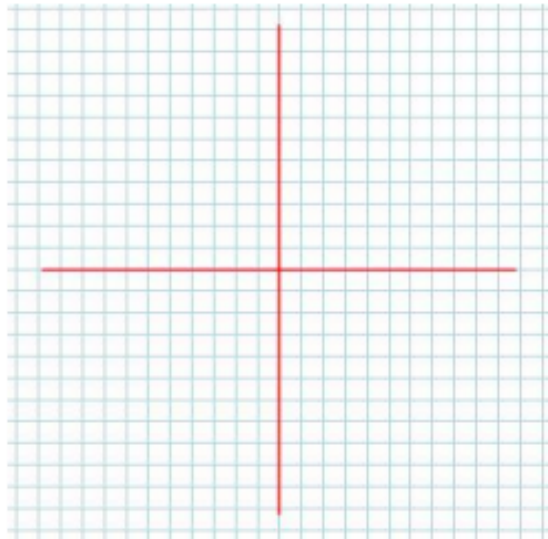
1)



2)



3)



End Behaviour of Functions

Here we are concerned with how the function is behaving as x gets

As x gets (which we write $x \rightarrow \infty$, or $x \rightarrow -\infty$)

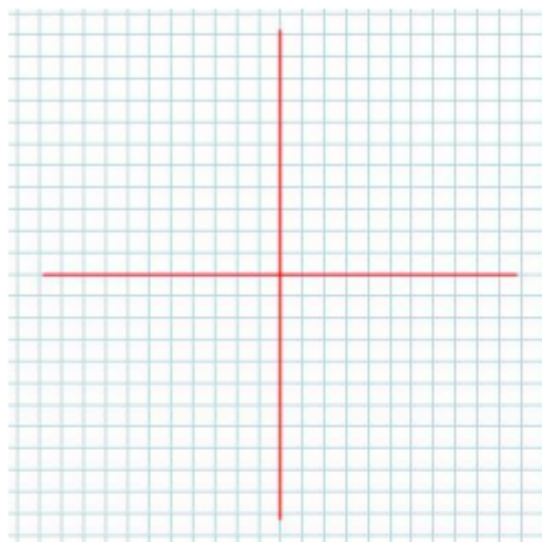
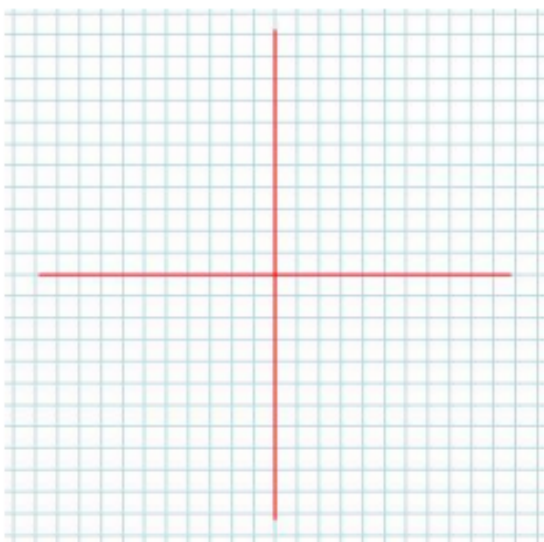
the functional values (for whatever function we are studying) can do one of three things:

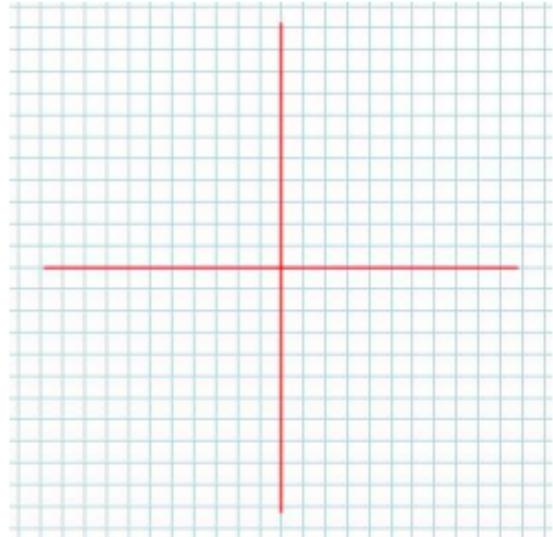
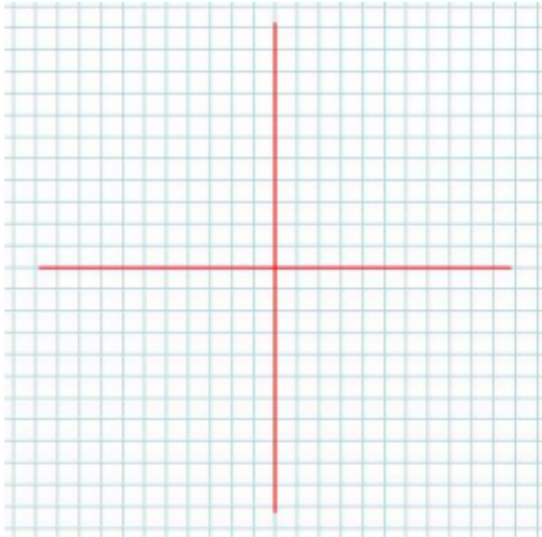
1) e.g. as $x \rightarrow \pm \infty$, $f(x) \rightarrow$

2)

3)

Pictures:



**Success Criteria**

- I can identify types of functions based on their graphical characteristics
- I can use different characteristics (intervals of increase/decrease, odd/even, end behaviour, continuity) to help me identify types of functions