1.4 Inverses of Functions

Learning Goal: We are learning to determine the equation of an inverse relation and the conditions for an inverse relation to be a function.

The inestimable William Groot has a saying:

An Inverse *Relation* is an UNDO

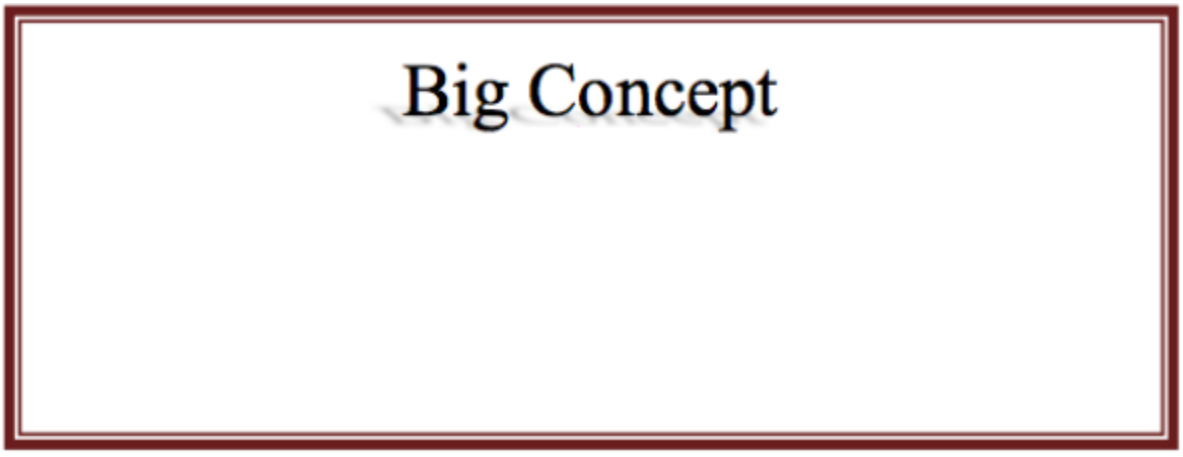
Definition 1.4.1

A **relation** is simply an algebraic relationship between domain values and range values.

Note: All functions are relations, but not all relations are functions

e.g. $x^2 + y^2 = 25$ is a relation, but it is not a function (it's a circle and so doesn't pass the VLT)

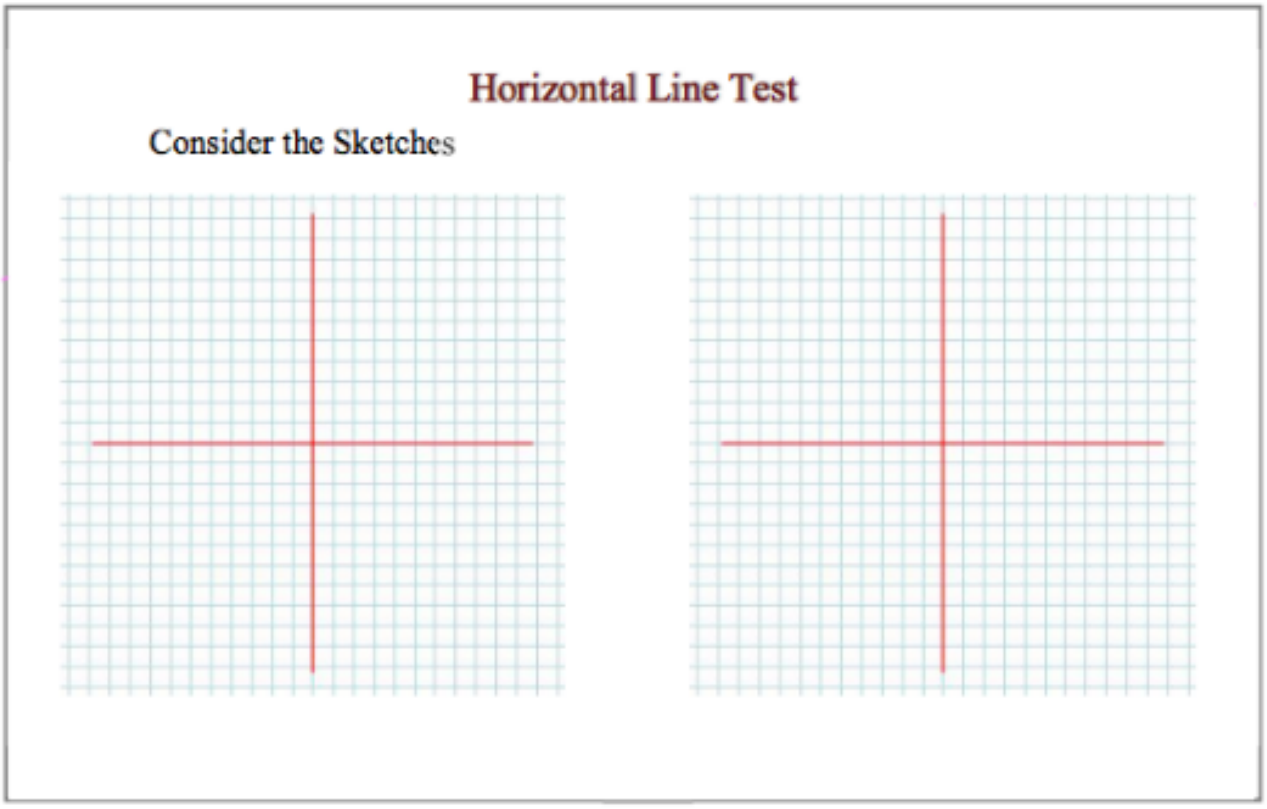
Consider the Arrow Diagram:



Example 1.4.1

Given the graph of $f(x)$ determine: D_f, R_f, $f^{-1}(x)$, $D_{f^{-1}}$, $R_{f^{-1}}$

$$f(x) = \{(2,3),(4,2),(5,6),(6,2)\}$$

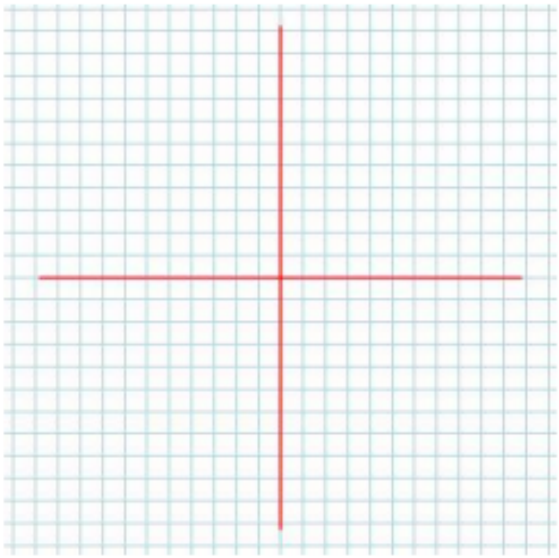


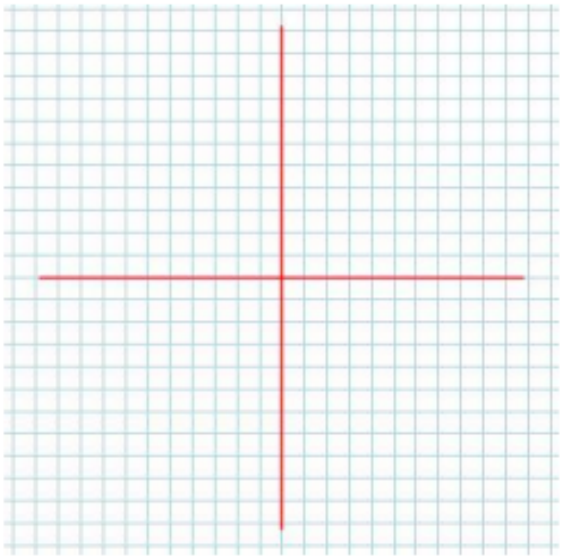
Determining the Inverse of a Function

We can determine the inverse of some given function in either of two ways: Graphically and Algebraically.

Note: Finding a function inverse graphically is not a very useful method, but it can be instructive.

Function Inverses Graphically





Restricting the Domain

Function Inverses Algebraically

Determining algebraic representations of inverse relations for given functions can be done in (at least) two ways:

1) Use algebra in a "brute force" manner (keeping in mind the Big Concept)
2) Use Transformations (keeping in mind "inverse operations")

Example 1.4.2

Determine the inverse of $f(x) = 2\sqrt{\frac{1}{3}(x - 1)} + 2$

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Here we will use "brute force".
Method:
1) Switch x and $f(x)$, and call "$f(x)$", $f^{-1}(x)$.
2) Solve for $f^{-1}(x)$

State the domain and range of both the function and its inverse.

Example 1.4.3

Using transformations determine the inverse of $f(x) = 2\sqrt{\frac{1}{3}(x - 1)} + 2$.

Example 1.4.4

Determine the inverse of $g(x) = -2(x-1)^2 + 3$.

Note that the natural domain of $g(x)$ is $(-\infty, \infty)$. However, $g(x)$ does not pass the HLT so its inverse is not a function. Determine a restricted domain for $g(x)$ so that $g^{-1}(x)$ ***is*** a function.

Example 1.4.5

Given $f(x) = kx^2 - 3$ and given $f^{-1}(5) = 2$, find k.

Success Criteria

- I can determine the equation of an inverse function using various methods
- I can determine whether an inverse relation is a function, and whether or not the domain needs to be restricted