1.5 Piecewise Defined Functions

Learning Goal: We are learning to understand, interpret, and graph situations that are described by piecewise functions; and learning the properties of the absolute value function.

Some aspects of "reality" exhibit different (as opposed to changing)

To capture those different mathematically may require using different

over different of the domain.

Absolute Value

Before discussing piecewise defined functions in general, we will first review the concept of *absolute value.*

Definition 1.5.1

The absolute ***value*** of a number, *x*, is given by

e.g.'s

$|22| =$

$|-8| =$

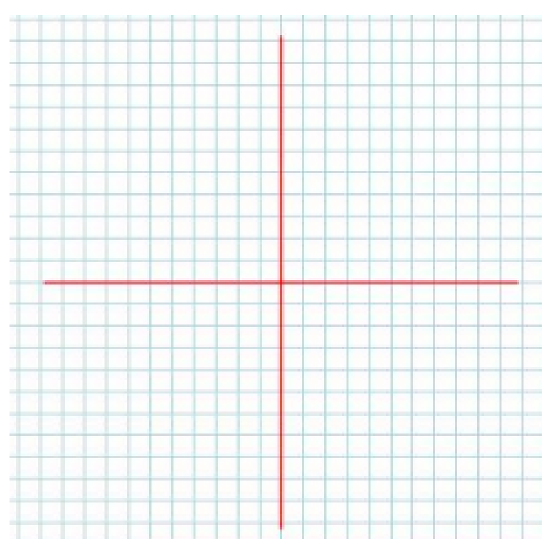
$|8-13| =$

Absolute Value Functions

We can define the function which returns the absolute value for any given number as

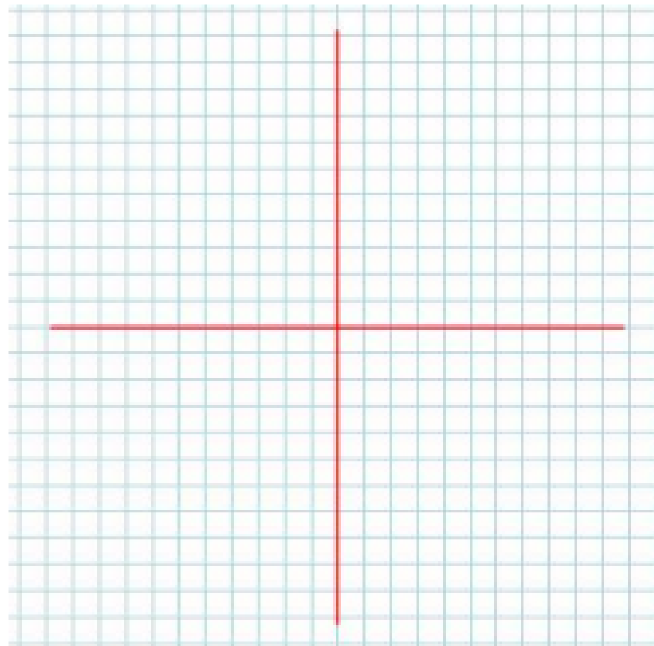
$f(x) = |x| =$ Picture

(Two behaviours!)



We can go further and define functions which return the absolute value for more complicated expressions.

e.g. Sketch $g(x) = |x^2 - 1|$ (note: $g(x)$ takes the absolute value of the ***functional values*** for the "basic" function $f(x) = x^2 - 1$)



(Three functional behaviours)

Absolute Value and Domain Intervals (and Quadratic Inequalities)

e.g.'s Sketch the solution sets of the following inequalities:

a) $x > 1$

b) $x \leq 2$

c) $1 < x \leq 4$

d) $-2 < x < 2$

Note the symmetry in part d)! Sometimes it's useful to think of absolute value as

Using the above notion we can thus use absolute value to denote the interval $-2 < x < 2$ as

e.g. Solve the quadratic equation

$$x^2 = 4$$

e.g. Solve the quadratic inequalities, and sketch the solution sets:

a) $x^2 < 4$

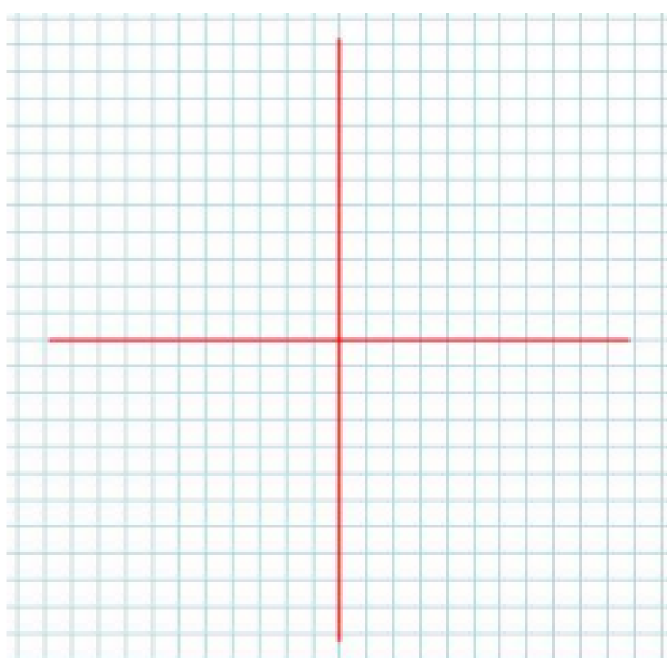
b) $x^2 \geq 3$

And now we return our attention to general **Piecewise Defined Functions**

Example 1.5.1

You are saving for university, and place $1000 into a sock every six months. After 18 months you wake up and put the money in your sock into an interest bearing bank account. You continue making deposits. Give a graphical representation of this situation.

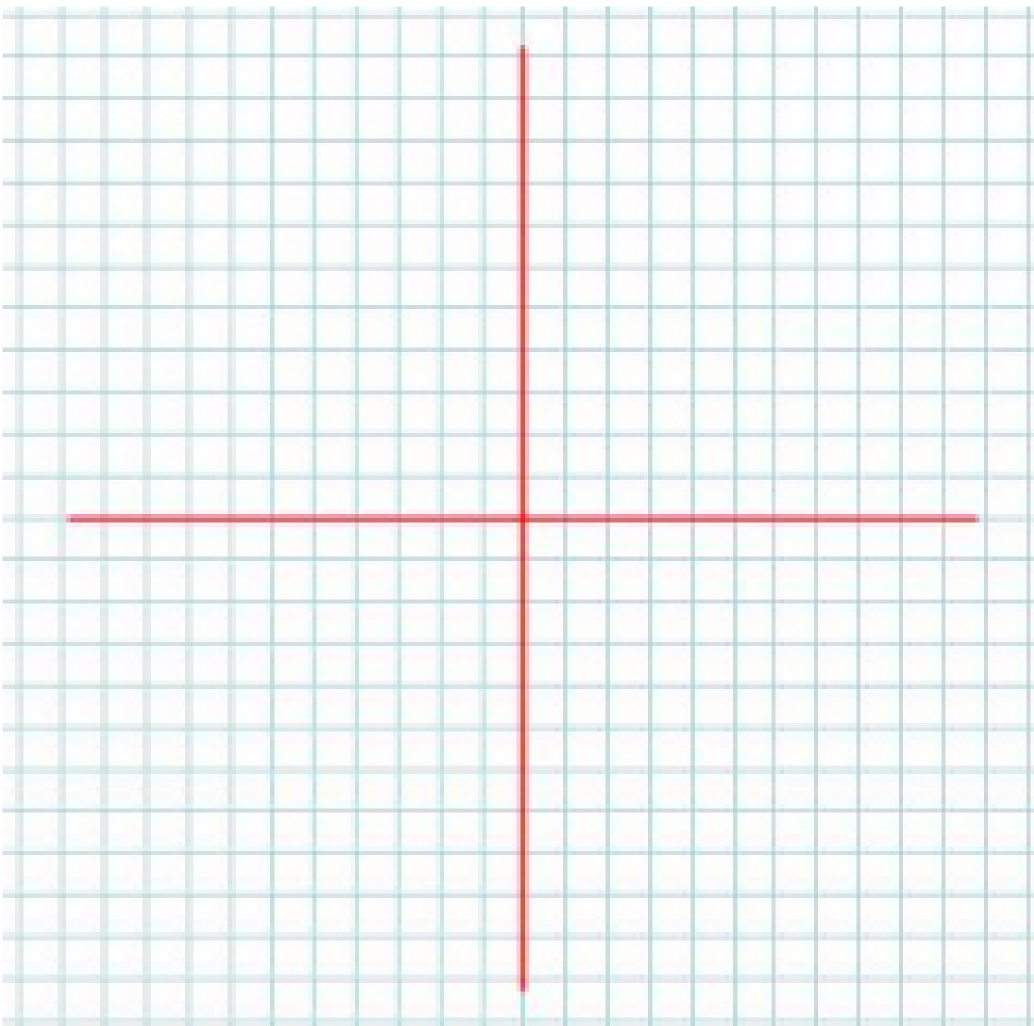
What is the behaviour of the amount of money you have saved? How is the behaviour changing?



Example 1.5.2

Note the notation we use for piecewise defined functions. Each functional behaviour has a mathematical representation, Determine the graphical representation for:

$$f(x) = \begin{cases} x^2, & x \in [-1,2) \\ 2, & x \in [2,3] \\ x+1, & x \in (3,\infty) \end{cases}$$



Example 1.5.3

Determine a possible algebraic representation which describes the given functional behaviour.

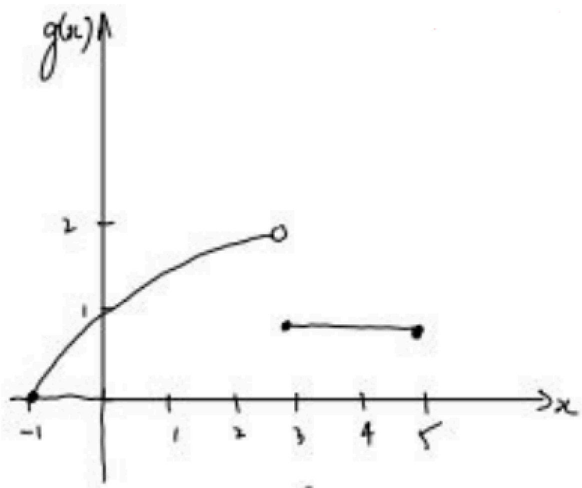


Figure 1.5.3

Success Criteria

- I can absolutely understand the absolute value function
- I can graph the piecewise function by graphing each piece over the given interval
- I can determine if a piecewise function is continuous or not