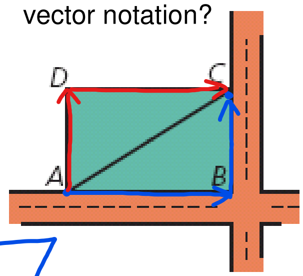
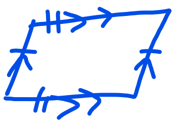
Vector Addition/Subtraction & Properties

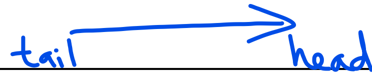
1. Suppose rectangle ABCD is a park at a corner of an intersection. What are two ways to get from A to C written in vector notation?



$\vec{AC} = \vec{AD} + \vec{DC}$

$\vec{AC} = \vec{AB} + \vec{BC}$



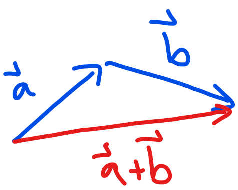


Adding Vectors RULE

$\vec{AC} = \vec{AX} + \vec{XC}$

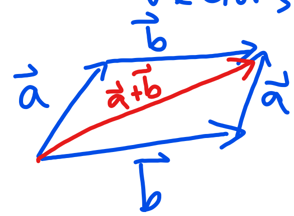
Triangle Law of Vector Addition.

Place vectors "head to tail"



Parallelogram Law of Vector Addition.

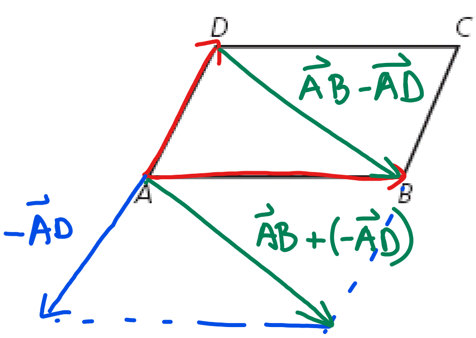
place vectors "tail to tail"



$10 - 5 = 10 + (-5)$

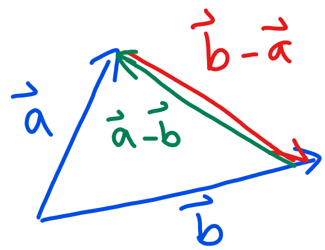
2. In parallelogram ABCD, find the difference $\overrightarrow{AB} - \overrightarrow{AD}$
   a. geometrically
   b. algebraically

$\vec{AB} + (-\vec{AD})$



Vector Subtraction.

Place vectors tail to tail



$(2+5)+3$
$7+3$
10

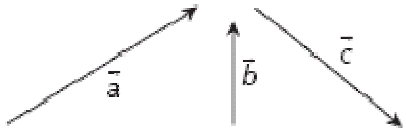
$2+(5+3)$
$2+8$
10

$2+5=5+2=7$

$a+b=b+a$

3. Show a <u>geometric proof</u> of the associative law.

eg.



$3(x+1)=3x+3$

$x(x+1)=x^2+x$

Commutative Law of Addition.

$\vec{u}+\vec{v}=\vec{v}+\vec{u}$

Associative Law of Addition.

$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$

Distributive properties

$k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$

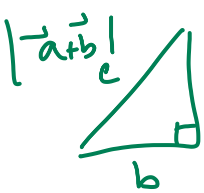
$(m+n)\vec{a}=m\vec{a}+n\vec{a}$

Another Associative property

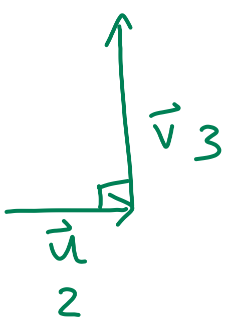
$mn\vec{a}=m(n\vec{a})=(mn)\vec{a}$

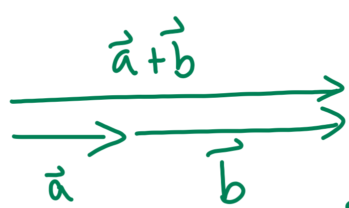
$\left.\begin{matrix}k\\m\\n\end{matrix}\right\}$ Scalar

4. Show an <u>informal proof</u> of the triangle inequality: $|\vec{u}+\vec{v}|\le|\vec{u}|+|\vec{v}|$. When does equality hold?



less than





$\}\ |\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|$

Same direction

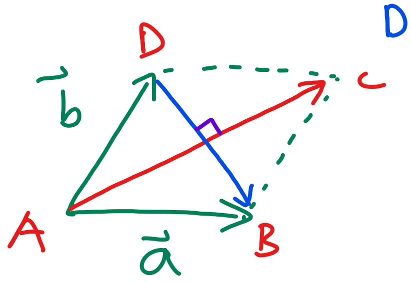
↳ Parallel

5. Show a formal proof that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular when $|\vec{a}|=|\vec{b}|$.

$|\vec{a}| = |\vec{b}|$

$\vec{AC} = \vec{a}+\vec{b}$

$\vec{DB} = \vec{a}-\vec{b}$



Unit Vector $(\hat{u})$

has magnitude of 1

$\hat{u} = \frac{\vec{u}}{|\vec{u}|}$

Parallel

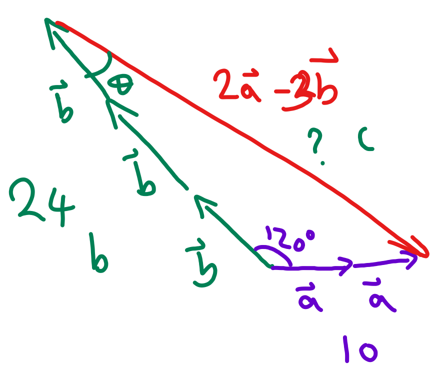
Collinear Vectors

Vectors that can be moved and place on 1 line

6. If $|\vec{a}|=5$, $|\vec{b}|=8$ and the angle between the two vectors is 120°.

a. Calculate the vector $2\vec{a}-3\vec{b}$

b. Determine the unit vector in the same direction as $2\vec{a}-3\vec{b}$



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a) Cosine law:

$c^2 = a^2+b^2-2ab\cos 120°$

$|2\vec{a}-3\vec{b}|^2 = |2\vec{a}|^2+|3\vec{b}|^2-2|2\vec{a}||3\vec{b}|\cos 120°$

$= 10^2+24^2-2(10)(24)\cos 120°$

$= 916$

$|2\vec{a}-3\vec{b}| = \sqrt{916} \rightarrow 2\sqrt{229}$

$\doteq 30.3$

$\frac{\sin\theta}{10} = \frac{\sin 120°}{30.3}$

$\theta = \sin^{-1}\left(\frac{10\sin 120°}{30.3}\right)$

$\therefore 2\vec{a}-3\vec{b}$ is 30.3 [17° off of $\vec{b}$] / 17°

b) unit vector $= \frac{2\vec{a}-3\vec{b}}{|2\vec{a}-3\vec{b}|}$

$= \frac{1}{30.3}(2\vec{a}-3\vec{b})$

or $\frac{1}{2\sqrt{229}}(2\vec{a}-3\vec{b})$