6.6 & 6.7 - Operations with Vectors in R2 and R3

Tuesday, October 28, 2025 7:22 PM

1. Find a single vector equivalent to each of the following

   a. $-\frac{1}{2}(4,-6,8)+\frac{3}{2}(4,-6,8)$ $=(-2,3,-4)+(6,-9,12)=(4,-6,8)$

   b. $5(9\hat{i}-7\hat{j})-5(-9\hat{i}+7\hat{k})$

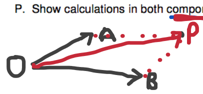
   c. If $\vec{a}=(2,-1,4)$ and $\vec{b}=3\hat{i}+8\hat{j}-6\hat{k}$ find $2\vec{a}-\vec{b}$ and its magnitude. $=3\sqrt{33}$

   $=(1,-10,14)$

(b) $-90\hat{i}-35\hat{j}-35\hat{k}$

2. If $A(1,-5,2)$ and $B(-3,4,4)$ are opposite vertices of parallelogram OAPB and O is the origin, find the coordinates of P. Show calculations in both component form and unit vector form.

$(0,0,0)$ $P(x,y,z)$



$\vec{OP}=\vec{OA}+\vec{OB}$

Component form

$=(1,-5,2)+(-3,4,4)$

$=(-2,-1,6)$

$1\hat{i}+(-5\hat{j})+2\hat{k}+(-3\hat{i})+4\hat{j}+4\hat{k}$

Unit vector form

$=-2\hat{i}-\hat{j}+6\hat{k}$

3. Using vectors, demonstrate that the three points $A(5,-1)$, $B(-3,4)$ and $C(13,-6)$ are collinear.

when we move them they are on the same line. scalar multiple

$\vec{AB}=(-3-5,\ 4-(-1))$

$=(-8,5)$

$\vec{AC}=(13-5,\ -6-(-1))$

$=(8,-5)$

$\times -1$ k

Since $\vec{AB}=k\vec{AC}$ (scalar multiple) they are collinear.

$\hat{u}=\frac{\vec{u}}{|\vec{u}|}$ $\vec{u}=(x,y,z)$

4. Find the components of the unit vector with the direction opposite to $\vec{XY}$ where $X(7,4,-2)$ and $Y(1,2,1)$.

$\times -1$

$\vec{XY}=(1-7,\ 2-4,\ 1-(-2))$

$=(-6,-2,3)$ $\times -1$ $(6,2,-3)$

$|\vec{XY}|=\sqrt{-6^2+-2^2+3^2}$

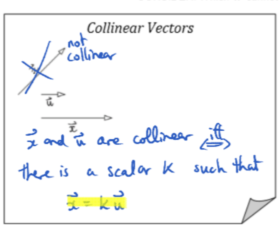
$=\sqrt{49}$

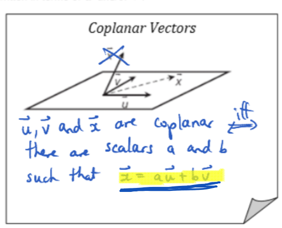
$=7$

$\frac{1}{7}(6,2,-3)$

Linear combinations are expressions $a\vec{u}+b\vec{v}$ where a,b are scalars and $\vec{u},\vec{v}$ are vectors ex. $2\hat{i}-3\hat{j}$ is a linear combination of $\hat{i}=(1,0)$ $\hat{j}=(0,1)$

CONSIDER: Which $\vec{x}$ cannot be written in terms of $\vec{u}$ and/or $\vec{v}$?





Spanning sets smallest # of vectors that can generate any other vector in the given space.

Spanning set for 1D R — (1D) - any vector will span a LINE

Spanning set for 2D R^2 — (2D) - any two noncollinear vectors will span a plane

Spanning set for 3D R^3 — (3D) - any three non-coplanar vectors will span a space

1. Explain what two vectors can span then determine if the following $\vec{x}$ and $\vec{u}$ are collinear.

   a. $\vec{x}=4\hat{i}-8\hat{j}$ $=(4,-8)$
   $\vec{u}=6\hat{i}-12\hat{j}$ $=(6,-12)$ $\times\frac{2}{3}$
   Yes

   b. $\vec{x}=(10,-8,3)$
   $\vec{u}=(5,-4,6)$ ~~$\times 2$~~

Not collinear
Because they are not scalar multiple

of each other
Span a plane in R3

a) span a line in R2

2. Explain what three vectors can span then determine if the following three vectors are coplanar.

R^3

a. $\vec{u} = (3,-1,4)$, $\vec{v} = (6,-4,-8)$, $\vec{w} = (7,-3,4)$

b. $\vec{u} = (1,3,2)$, $\vec{v} = (1,-1,1)$, $\vec{w} = (5,1,-4)$

$\vec{u} = a\vec{v} + b\vec{w}$ ✓

$(3,-1,4) \stackrel{?}{=} a(6,-4,-8) + b(7,-3,4)$

$(3,-1,4) = (6a,-4a,-8a) + (7b,-3b,4b)$

$3 = 6a + 7b$ ① ×2 $\quad -3+6 = 14b - 9b$ $\quad -1 = -4a - 3(3/5)$

$-1 = -4a - 3b$ ② ×3 $\quad 3 = 5b$ $\quad -4a$

$4 = -8a + 4b$ ③ $\quad \frac{3}{5} = b$ $\quad \frac{4}{5} = -4a$

$a = -1/5$

check $4 \stackrel{?}{=} -8\left(-\frac{1}{5}\right) + 4\left(\frac{3}{5}\right)$

✓ 4

Not coplanar.

Therefore, they are coplanar