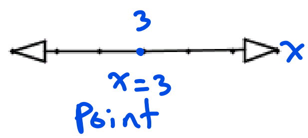
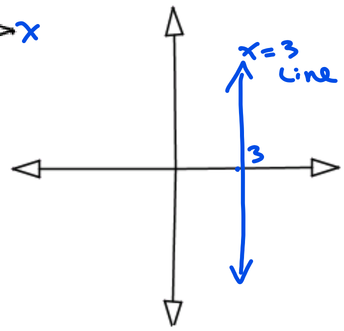
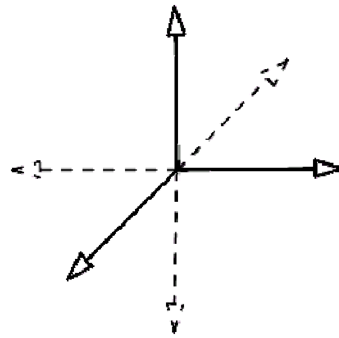
Vectors in R^2 and R^3 - ALGEBRAIC vectors

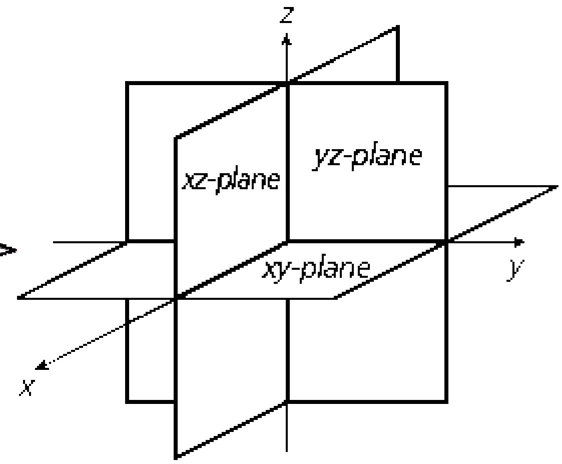
↳ Cartien plane — Coordinates (x, y)

R – line 1D **R^2 – plane** 2D **R^3 – space**

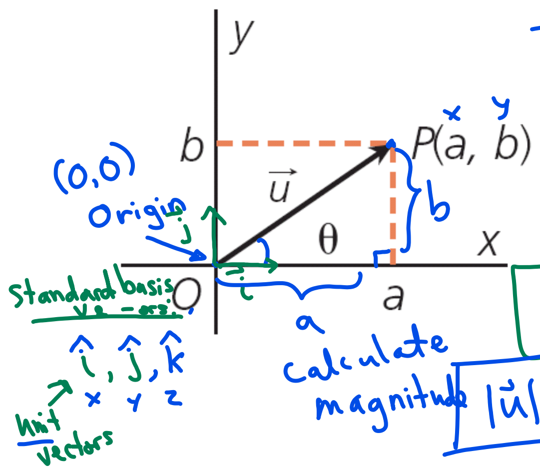








Position Vector:
- is a vector has the tail at the origin of the coordinate system

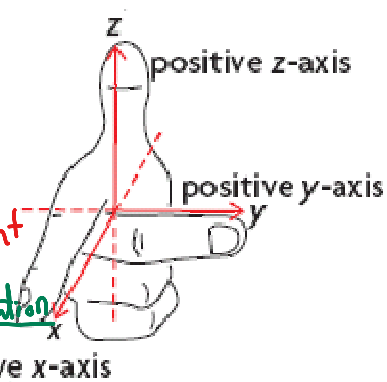


$\overrightarrow{OP} = \vec{u} = \boxed{(a, b)}$ ← Component form (3, 4)

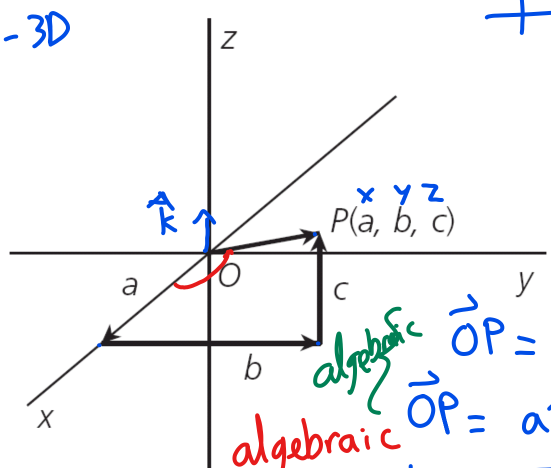
$\boxed{\vec{u} = a\hat{i} + b\hat{j}}$ ← linear combination of standard basis vectors.

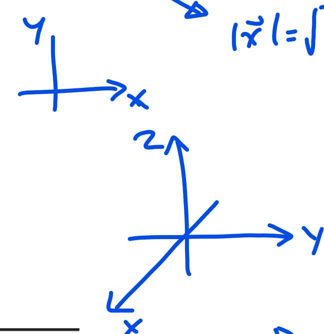
calculate magnitude $\boxed{|\vec{u}| = \sqrt{a^2 + b^2}}$

ex: $\vec{x}(3, 4)$
$|\vec{x}| = \sqrt{3^2 + 4^2}$



R^3 - 3D

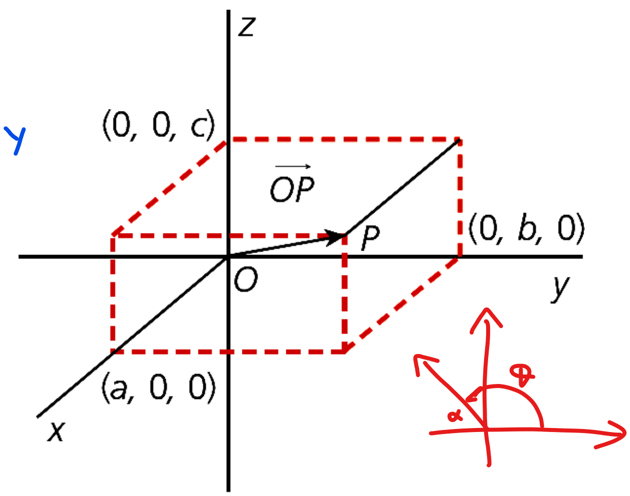




algebraic $\overrightarrow{OP} = (a, b, c)$

algebraic $\overrightarrow{OP} = a\hat{i} + b\hat{j} + c\hat{k}$

$|\overrightarrow{OP}| = \sqrt{a^2 + b^2 + c^2}$



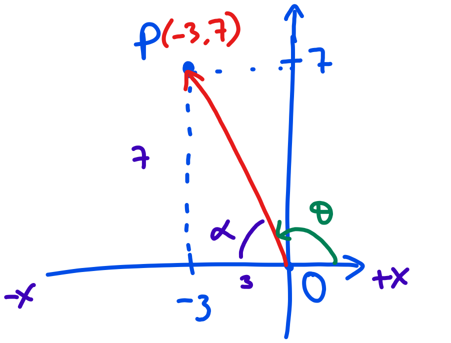
Geometric Form: must be from positive x-axis

2D: $\vec{u} = |\vec{u}|\,[\theta \text{ direction}]$

3D: $\vec{u} = |\vec{u}|\,[\alpha, \beta, \gamma]$
x-axis, y-axis, z-axis

$\left(\frac{x}{r}, \frac{y}{r}\right)$

1. Draw a position vector of the point P(– 3, 7) then
   a. express it in both algebraic vector notations AND in geometric notation
   ~~b.~~ find the unit vector, how does it tie to unit circles you've learned in grade 11/12?



$\tan\alpha = \frac{7}{3}$

$\alpha = \tan^{-1}\left(\frac{7}{3}\right)$

$\doteq 67^\circ$

Algebraic Notations:

↳ component form $\vec{OP} = (-3, 7)$

↳ linear combination $\vec{OP} = -3\hat{i} + 7\hat{j}$

Geometric Notation:

2D: $\vec{u} = |\vec{u}|\ [\theta]$

$|\vec{OP}| = \sqrt{(-3)^2 + (7)^2} = \sqrt{58}$

$\theta = 180^\circ - 67^\circ = 113^\circ$

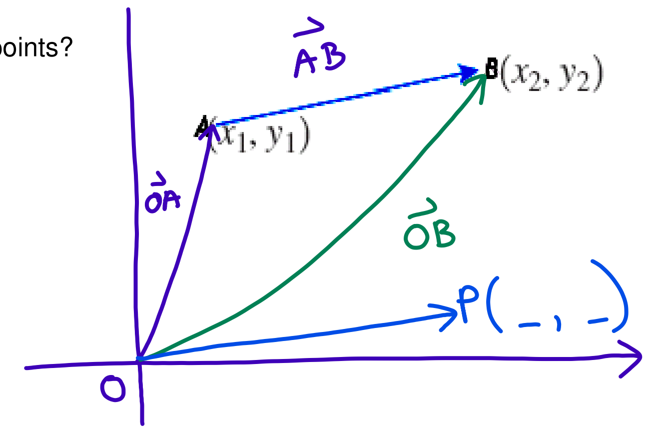
$\vec{OP} = \sqrt{58}\ [113^\circ]$

2. How do you find the related position vector of any vector between points?

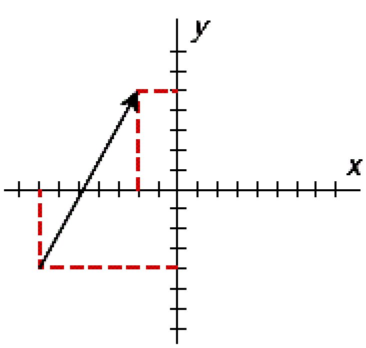
$\vec{OP} = \vec{AB} = \vec{OB} - \vec{OA}$

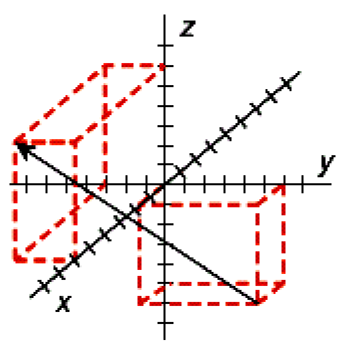
$= (x_2, y_2) - (x_1, y_1)$

$= (x_2 - x_1, y_2 - y_1)$



~~3.~~ Reposition each of the following vectors so that its initial point is at the origin, and determine its components.

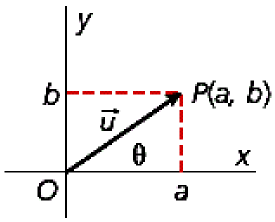




Formulas can be rewritten to use any vector between two point coordinates

$A(x_1, y_1)B(x_2, y_2)$ $A(x_1, y_1, z_1)B(x_2, y_2, z_2)$

Vectors in R^2



$\vec{AB} = (x_2 - x_1, y_2 - y_1)$

$|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

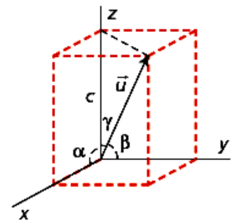
geometric: $\vec{AB} = |\vec{AB}|\,[\theta]$

unit vector: $\hat{AB} = \dfrac{\vec{AB}}{|\vec{AB}|}$ ① $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$\hat{AB} = (\cos\theta, \sin\theta)$

② $\hat{a} =$

Vectors in R^3



$\vec{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

$|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

geometric: $\vec{AB} = |\vec{AB}|\,[\alpha, \beta, \gamma]$

Unit vector: $\hat{AB} = \dfrac{\vec{AB}}{|\vec{AB}|} = (\cos\alpha, \cos\beta, \cos\gamma)$

(a, b)

$\vec{a} = |\vec{a}|\,\hat{a}$

$= |\vec{a}|(\cos\theta, \sin\theta)$

4. Express as a vector in component form

a. $|\vec{a}| = 12$, $\theta = 330^o$ in R^2

b. $|\vec{u}| = 8$, $\alpha = 60^o$, $\beta = 150^o$ in R^3 $(4, -4\sqrt{3}, 0)$

$\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

a) $\vec{a} = |\vec{a}|\,[\theta]$ Geo.

$\vec{a} = 12\,[330^\circ]$

unit vector

$\vec{a} = |\vec{a}|\,\hat{a}$

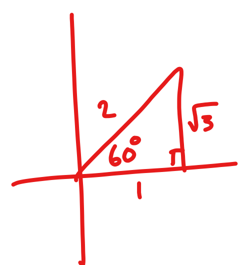
$= 12\,(\cos 330^\circ, \sin 330^\circ)$

$= 12\left(\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$

$= (6\sqrt{3}, -6)$

$\vec{a} = (a, b, c)$

$|\vec{a}| = \sqrt{a^2 + b^2 + c^2}$



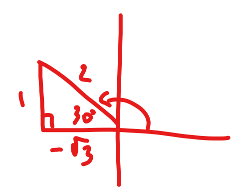
b) ① Find $\gamma = ?$

$\hat{u} = (\overset{a}{\cos\alpha}, \overset{b}{\cos\beta}, \overset{c}{\cos\gamma})$

$|\hat{u}| = \sqrt{(\cos\alpha)^2 + (\cos\beta)^2 + (\cos\gamma)^2}$

$1 = \sqrt{(\cos 60^\circ)^2 + (\cos 150^\circ)^2 + (\cos\gamma)^2}$

$a = \cos x$



$1 = \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{-\sqrt{3}}{2}\right)^2 + \cos^2\gamma$

$0 = \cos^2\gamma$

$0 = \cos\gamma$

$\gamma = 90^\circ$

② $\vec{u} = |\vec{u}|\,\hat{u}$

$= 8\,(\cos 60^\circ, \cos 150^\circ, \cos\gamma)$

$= 8\left(\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}, 0\right)$

$= (4, -4\sqrt{3}, 0)$

+

−

Operations with Vectors in R^2 and R^3

Scalar multip.

1. Find a single vector equivalent to each of the following

   a. $-\frac{1}{2}(4,-6,8)+\frac{3}{2}(4,-6,8)$

   b. $5(9\hat{i}-7\hat{j})-5(-9\hat{i}+7\hat{k})$

   c. If $\vec{a}=(2,-1,4)$ and $\vec{b}=3\hat{i}+8\hat{j}-6\hat{k}$ find $2\vec{a}-\vec{b}$ and its magnitude.

$\vec{b}=(3,8,-6)$

a) $=(-2,3,-4)+(6,-9,12)$
$=(4,-6,8)$

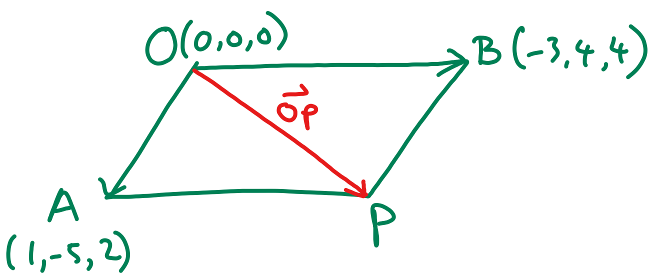
b) $(45\hat{i}-35\hat{j})-(-45\hat{i}+35\hat{k})$
$=90\hat{i}-35\hat{j}-35\hat{k}$

c) $2\vec{a}-\vec{b}=2(2,-1,4)-(3,8,-6)$
$=(1,-10,14)$ Component.
$=\hat{i}-10\hat{j}+14\hat{k}$ linear comb.
$|2\vec{a}-\vec{b}|=\sqrt{1^2+(-10)^2+14^2}=3\sqrt{33}$

2. If $A(1,-5,2)$ and $B(-3,4,4)$ are opposite vertices of parallelogram OAPB and O is the origin, find the coordinates of P. Show calculations in both component form and unit vector form.

Point/corner

linear comb.

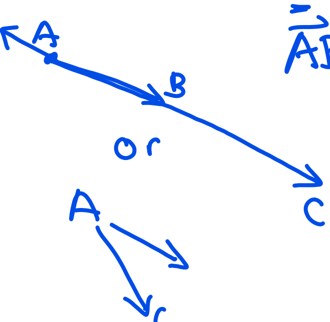


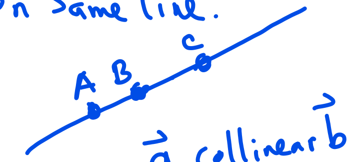
$\vec{OP}=\vec{OA}+\vec{OB}$
$=(1,-5,2)+(-3,4,4)$
$=(-2,-1,6)\rightarrow$ Component form
$\vec{OP}=-2\hat{i}-1\hat{j}+6\hat{k}\rightarrow$ unit vector form

3. Using vectors, demonstrate that the three points $A(5,-1), B(-3,4)$ and $C(13,-6)$ are collinear.

↔ On same line.

$\vec{AB}=(-3-5,\ 4-(-1))$ $\quad$ $\vec{AC}=(13-5,-6-(-1))$
$\vec{AB}=(-8,5)$ $\quad$ $\vec{AC}=(8,-5)$





$\vec{a}$ collinear $\vec{b}$
$\rightarrow \vec{a}=k\vec{b}$

$\times -1$

$\vec{AB}=-1(\vec{AC})$ $\quad \therefore$ They are collinear.

4. Find the components of the unit vector with the direction opposite to $\vec{XY}$ where $X(7,4,-2)$ and $Y(1,2,1)$.

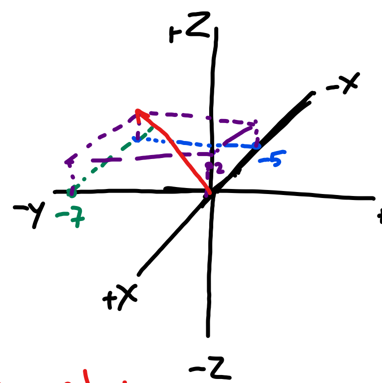
$\hat{xy}=\frac{\vec{xy}}{|\vec{xy}|}$ $\quad$ $-1(\vec{XY})$

① Find $\vec{XY}=(\ \ ,\ \ ,\ \ )$ $\quad$ $\vec{XY}=(1,2,1)-(7,4,-2)$ (Y, X)

② Find $-\vec{XY}$ $\quad$ $\frac{(6,2,-3)}{7}$

③ Find $|\vec{XY}|$ $\quad$ ④ $-\hat{XY}=\frac{1}{|\vec{XY}|}(-\vec{XY})$ $\quad$ $\boxed{\frac{1}{7}(6,2,-3)}$

5. Draw a position vector of the point T(– 5, – 7, 2) then
   a. find a unit vector in the same direction as $\overrightarrow{OT}$
   b. express it in two algebraic vector notations AND geometric notation



a) $\hat{OT} = \frac{1}{|\overrightarrow{OT}|}(\overrightarrow{OT})$ $|\overrightarrow{OT}| = \sqrt{78}$

$= \frac{1}{\sqrt{78}}(-5,-7,2)$

$\frac{-5}{\sqrt{78}} = \cos\alpha$

$\alpha = 124^\circ$

Algebraic component $(-5,-7,2)$

linear comb. $-5\hat{i} - 7\hat{j} + 2k$

$\hat{OT} = (\cos\alpha, \cos\beta, \cos\gamma)$

$\left(\frac{-5}{\sqrt{78}}, \frac{-7}{\sqrt{78}}, \frac{2}{\sqrt{78}}\right) = (\cos\alpha, \cos\beta, \cos\gamma)$

Geometric:

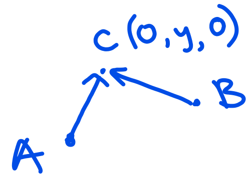
$\overrightarrow{OT} = |\overrightarrow{OT}|[\alpha = \ , \beta = \ , \gamma = \ ]$

$\overrightarrow{OT} = \sqrt{78}\,[124^\circ, 142^\circ, 77^\circ]$

A B

5. Find the point on the y-axis that is equidistant from the points (2, – 1, 1) and (0, 1, 3)

same distance



$|\overrightarrow{AC}| = |\overrightarrow{BC}|$

Pg. 326 #10

Pg. 332 #1, 2, 4, 8, 9, 11, 12

Journal Unit 1 #4, 5a.