Linear Combinations and Spanning Sets

R^3: $\hat{k}\ (0,0,1)$, $\hat{i}\ (1,0,0)$, $\hat{j}\ (0,1,0)$

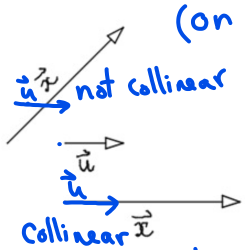
Linear combinations are expressions $a\vec{u} + b\vec{v}$ where a, b are scalars, $\vec{u}, \vec{v}$ are vectors

ex: $2\hat{i} - 3\hat{j}$ 2, −3 ⇒ scalars; $\hat{i}\ (1,0)$ $\hat{j}\ (0,1)$ R^2

CONSIDER: Which $\vec{x}$ cannot be written in terms of $\vec{u}$ and/or $\vec{v}$?

Collinear Vectors

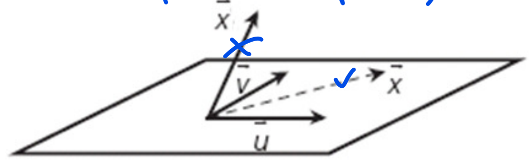
(on same line)



$\vec{x}$ and $\vec{u}$ are collinear iff ⟺ there is a scalar, k, such that $\vec{x} = k\vec{u}$

Coplanar Vectors

(on same plane)



$\vec{u}, \vec{v}$ and $\vec{x}$ are coplanar iff ⟺ there are scalars a and b such that $\boxed{\vec{x} = a\vec{u} + b\vec{v}}$

Spanning sets: Smallest # of vectors that can generate any other vector in a given space.

span ⇒ "make"

Spanning set for R: 1D – any vector will span a line

Spanning set for R^2: 2D – any two noncollinear vectors will span a plane

Spanning set for R^3: 3D – any three non-coplanar vectors will span a space

① R, R^2, R^3 ②

1. Explain what two vectors can span then determine if the following $\vec{x}$ and $\vec{u}$ are collinear.

a. $\vec{x} = 4\hat{i} - 8\hat{j}$

$\vec{u} = 6\hat{i} - 12\hat{j}$

$k = \times\frac{2}{3}$; $\vec{x} = (4, -8)$, $\vec{u} = (6, -12)$; $k = \times\frac{3}{2}$

$\vec{x} \overset{?}{=} k\vec{u}$ ✓

∴ Collinear

They can span a line in R^2

b. $\vec{x} = (10, -8, 3)$

$\vec{u} = (5, -4, 6)$

2 2 $\frac{1}{2}$

$\vec{x} \overset{?}{=} k\vec{u}$ ✗

∴ noncollinear

span a plane in R^3

Assume $\vec{x} = k\vec{u}$

$(10, -8, 3) = k(5, -4, 6)$

① $10 = 5k \Rightarrow k = 2$

② $-8 = -4k \Rightarrow k = 2$

③ $3 = 6k \Rightarrow k = \frac{1}{2}$

∴ $\vec{x} \neq k\vec{u}$

∴ noncollinear

2. Explain what three vectors can span then determine if the following three vectors are coplanar.

a. $\vec{u} = (3,-1,4)$, $\vec{v} = (6,-4,-8)$, $\vec{w} = (7,-3,4)$

b. $\vec{u} = (1,3,2)$, $\vec{v} = (1,-1,1)$, $\vec{w} = (5,1,-4)$

non-coplanar

Span a space in R^3

Review Solve by "elimination"

① $\vec{u} = a\vec{v} + b\vec{w}$? ✓

$(3,-1,4) = a(6,-4,-8) + b(7,-3,4)$

$(3,-1,4) = (6a,-4a,-8a) + (7b,-3b,4b)$

② ① $3 = 6a + 7b$

② $-1 = -4a - 3b$

③ $4 = -8a + 4b$

③ eliminate "a"

①×2 : $6 = 12a + 14b$

②×3 : $-3 = -12a - 9b$ +

$3 = 0 + 5b$

$b = \frac{3}{5}$

④ Sub $b = \frac{3}{5}$ into ①

$3 = 6a + 7\left(\frac{3}{5}\right)$

$6a = 3 - \frac{21}{5}$

$a = -\frac{1}{5}$

⑤ Check

Sub $a = -\frac{1}{5}$ and $b = \frac{3}{5}$ into ③

$Ls = 4$ ✓

$Rs = -8\left(-\frac{1}{5}\right) + 4\left(\frac{3}{5}\right)$

$= \frac{8}{5} + \frac{12}{5}$

$= \frac{20}{5}$

$= 4$ ✓

⑥ $\therefore \vec{u} = -\frac{1}{5}\vec{v} + \frac{3}{5}\vec{w}$

$\therefore$ They are coplanar and span a plane in R^3

Pg. 341 # 7b, 11, 13, 14