

Review Transformation of Functions



1. Describe what each constant in $y = af(k(x-d)) + c$ controls

a = Vertical stretch or compress
 $|a| > 1$ stretch, $0 < |a| < 1$ compress

Reflection in x -axis, $a < 0$ (negative)

k = Horizontal stretch or compress
 $0 < |k| < 1$ stretch, $|k| > 1$ compress

Reflection in y -axis, $k < 0$ (negative)

d = Horizontal shift left or right

$d < 0$ left, ex: $(x+3)$, $d = -3$
 $d > 0$ right, ex: $(x-3)$, $d = 3$

c = Vertical shift up or down
 $c > 0$ up, $c < 0$ down

3. Sketch each of the following

a. $h(x) = \frac{4}{6+0.5x} + 2$

Parent: $y = \frac{1}{x}$

Key: H.A. $y = 0$
 V.A. $x = -12$
 (1, 1), (-1, -1)

$a = 4$

$k = 0.5$

$d = -12$

$c = 2$

• Vertical stretch by a factor of 4

• Horizontal stretch by a factor of 2 ($\frac{1}{k} = \frac{1}{0.5} = 2$)

• Shift left 12 units

• Shift up 2 units

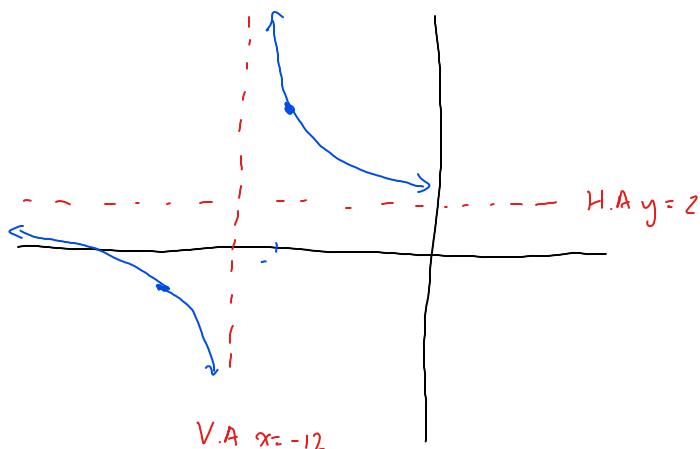
$(x, y) \rightarrow (2x-12, 4y+2)$

H.A. $y = 0 \rightarrow y = 2$

V.A. $x = 0 \rightarrow x = -12$

(1, 1) $\rightarrow (2(1)-12, 4(1)+2) \rightarrow (-10, 6)$

(-1, -1) $\rightarrow (2(-1)-12, 4(-1)+2) \rightarrow (-14, -2)$



2. Describe the most effective method of applying transformations.

① factor out "k" so we can see "d".
 $\hookrightarrow x$ has no number in front of it.

② use Mapping Rule to graph.

$(x, y) \rightarrow (\frac{x}{k} + d, ay + c)$

Key points:

x	y
1	2

$\frac{x}{2} + 3$	$3y + 7$
4	13

b. $i(x) = 2\sqrt{4-x} - 1$

Parent: $y = \sqrt{x}$

$i(x) = 2\sqrt{-1(x-4)} - 1$

(0, 0)
 (4, 2)
 (9, 3)

vertical stretch by a factor of 2
 horizontal reflection over y -axis
 shift right 4 units
 shift down 1 unit



Review of Inverses of Functions $f^{-1}(x)$ or $g^{-1}(x)$

1. Clarify the meaning of the words opposite, reciprocal and inverse.

Opposite → switch sign

ex: $+$ → $-$
 \times → \div

$x-4 \rightarrow -x+4$

Reciprocal → flip fraction ($\frac{a}{b}$)

ex: $\frac{a}{b} \rightarrow \frac{b}{a}$
 $\frac{1}{2} \rightarrow \frac{2}{1}$ or 2
 $\frac{2x}{1} \rightarrow \frac{1}{2x}$

Inverse → Undo operations.

ex: $+$ ↔ $-$
 \times ↔ \div
 $\sqrt{}$ ↔ squared
 $\sin()$ ↔ $\sin^{-1}()$ "sine inverse"

2. Find the algebraic inverse for each of the following functions $f^{-1}(x)=?$

a. $f(x) = 2x - 6$

① $y = 2x - 6$

② $x = 2y - 6$

③ $x + 6 = 2y$
 $\frac{x+6}{2} = y$

④ $f^{-1}(x) = \frac{x+6}{2}$

Steps:

① replace $f(x) = y$

② switch x & y

③ Solve for y .

④ Replace y as $f^{-1}(x)$

d. $f(x) = -x^2 + 2x - 9$

$f^{-1}(x) = \pm \sqrt{-(x+8)} + 1$

b. $f(x) = \frac{x}{x-3}$

$f^{-1}(x) = \frac{-3x}{1-x}$

$y = \frac{x}{x-3}$

$x = \frac{y}{y-3}$

$x(y-3) = y$

$xy - 3x = y$

$-3x = y - xy$

$-3x = y(1-x)$

$\frac{-3x}{1-x} = y$

c. $f(x) = -2(x+5)^2 + 8$

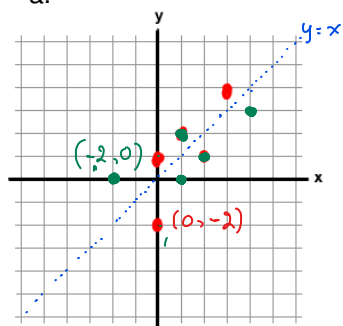
$f^{-1}(x) = \pm \sqrt{\frac{x-8}{-2}} - 5$

→ Reflection over the line $y = x$

3. Find the graphical inverse of each of the following



a.

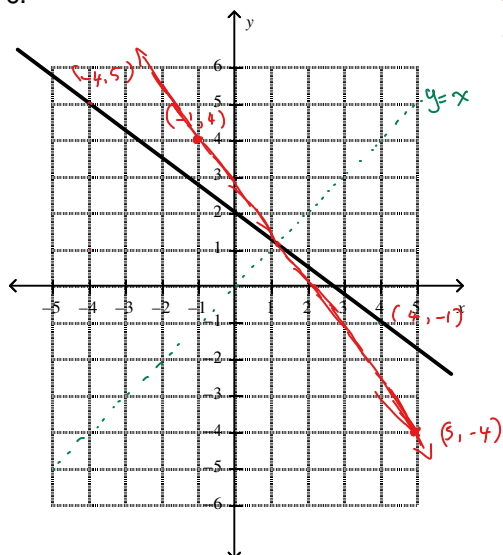


$$(x, y) \rightarrow (y, x)$$

$$(a, b) \rightarrow (b, a)$$

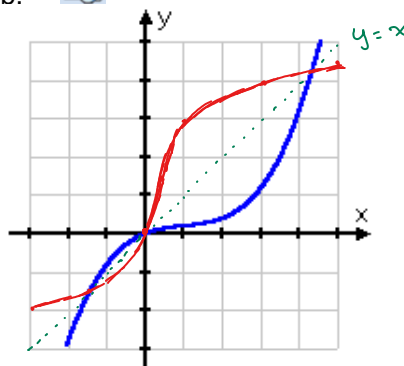
for point the $x \leftrightarrow y$
will switch for
inverse!

c.

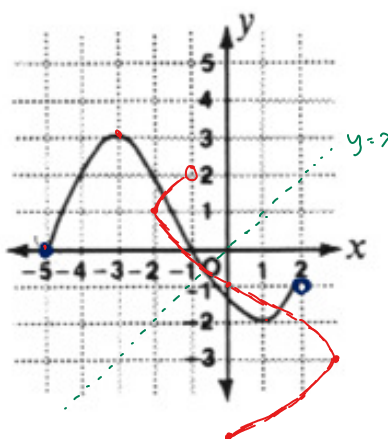


Point on the
line $y = x$
do not move!

b.



d.



How to Study:

- ① Read over note
- ② Write journal
- ③ Practice.

Homework:

- {Pg. 35 # 1, 3, 5
Pg. 43 # 3, 12

Journal Unit 1 # 2, 3

- e. Sketch $f(x) = -x^2 + 2x - 9$ (same question as 2d)
then sketch $f^{-1}(x)$ on the same grid



4. Can you think of applications of inverse functions?