New Properties of Functions

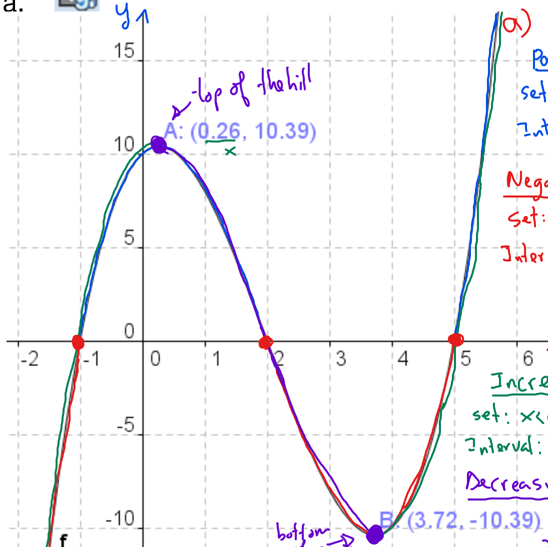
Set Notation $\{x \in \mathbb{R} \mid 3 \le x < 5\}$
Interval Notation $x \in [3, 5)$

1. Functions can be described using intervals. There are **positive and negative intervals**, as well as **increasing and decreasing intervals**. For the following graphs state both.

(above x-axis) + (below x-axis) – y values

slope is + slope is –

a.



y, x-axis, top of the hill, bottom of valley

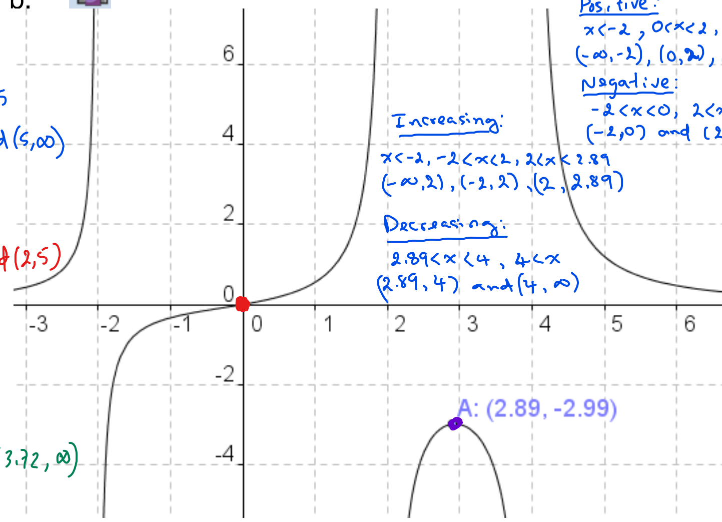
Positive Interval:
Set: $-1 < x < 2$ and $x > 5$
Interval: $x \in (-1, 2)$ and $(5, \infty)$

Negative Interval:
Set: $x < -1$ and $2 < x < 5$
Interval: $x \in (-\infty, -1)$ and $(2, 5)$

Increasing Interval:
set: $x < 0.26$ and $x > 3.72$
Interval: $x \in (-\infty, 0.26)$ and $(3.72, \infty)$

Decreasing Interval:
$0.26 < x < 3.72$
$x \in (0.26, 3.72)$

b.



Positive:
$x < -2$, $0 < x < 2$, $x > 4$
$(-\infty, -2)$, $(0, 2)$, $(4, \infty)$

Negative:
$-2 < x < 0$, $2 < x < 4$
$(-2, 0)$ and $(2, 4)$

Increasing:
$x < -2$, $-2 < x < 2$, $2 < x < 2.89$
$(-\infty, 2)$, $(-2, 2)$, $(2, 2.89)$

Decreasing:
$2.89 < x < 4$, $4 < x$
$(2.89, 4)$ and $(4, \infty)$

2. What are the points called that separate the positive and negative intervals?

x-intercepts / Zeros / Roots

3. What are the points called that separate the increasing decreasing intervals.

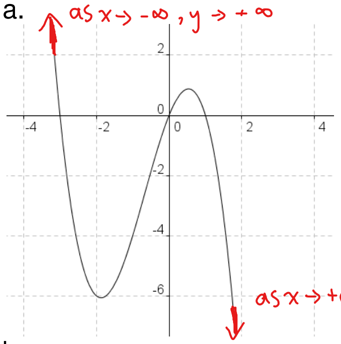
Turning / critical points

both left and right edge of the graph

4. Functions can also be decribed using **end behaviour.** This helps you recognize what the output values are approaching on the left-most and right-most sides of the graph. For the functions below state the end behaviour in proper notation.

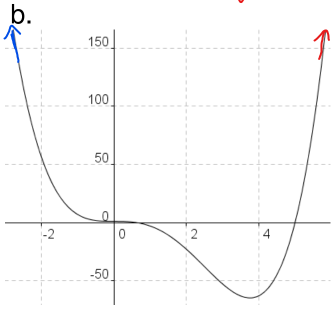
"→" "approach"

a.



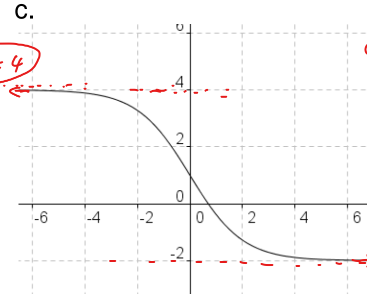
as $x \to -\infty$, $y \to +\infty$
as $x \to +\infty$, $y \to -\infty$

b.



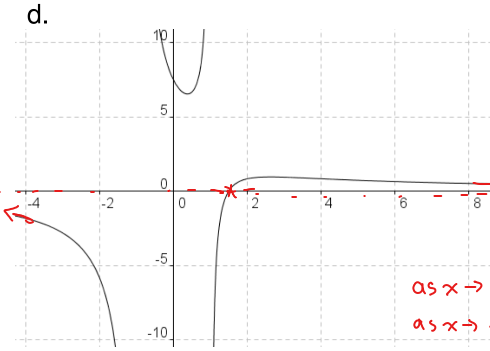
as $x \to -\infty$, $y \to +\infty$
as $x \to +\infty$, $y \to +\infty$

c.



H.A. $y = 4$
as $x \to -\infty$, $y \to 4$ ← H.A.
as $x \to +\infty$, $y \to -2$
H.A $y = -2$

d.



H.A $y = 0$
as $x \to -\infty$, $y \to 0$
as $x \to +\infty$, $y \to 0$

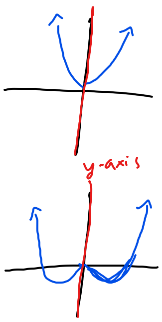
5. Vertical aymptotes cannot be crossed however notice in the last question the horizontal asymptote is crossed. How are horizontal asymptotes related to the end behaviour?

H.A shows end behaviour

6. Functions can be described using **symmetry**. Describe what is meant by **even**, **odd**, and **neither** symmetry. Show graphical representations and algebraic.

EVEN Symmetry:

Symmetry around y-axis
↳ Reflection in y-axis

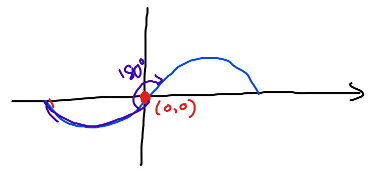


$\boxed{f(-x) = f(x)}$ *

↳ negative x into the equation

ODD Symmetry:
↳ Rotation symmetry of 180° around the origin (0,0)

↳ Reflection in both x and y-axis



$\boxed{f(-x) = -f(x)}$ *

7. Identify whether the following are odd, even or neither in their symmetry.

a. $y = 2x^3 - 5x$

sub $x = -x$

$2(-x)^3 - 5(-x)$

$= -2x^3 + 5x$

$= -(2x^3 - 5x)$

$= -y$ ∴ odd symmetry

b. $y = x^6 - 5x^4 + 2$

$(-x)^6 - 5(-x)^4 + 2$

$= x^6 - 5x^4 + 2$

$= y$ Even

c. $y = 4x^5 - x^3 + 10$

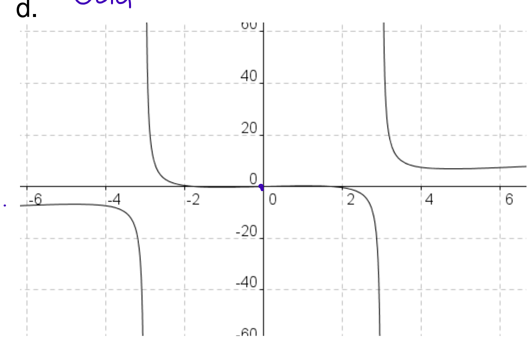
$4(-x)^5 - (-x)^3 + 10$

$= -4x^5 + x^3 + 10$

$= -(4x^5 - x^3 - 10)$

$\neq -y$ or y ∴ Neither

d. Odd



e. $y = \sqrt{4 - x^3}$

$\sqrt{4 - (-x)^3}$

$= \sqrt{4 + x^3}$

$\neq -y$ or y ∴ Neither

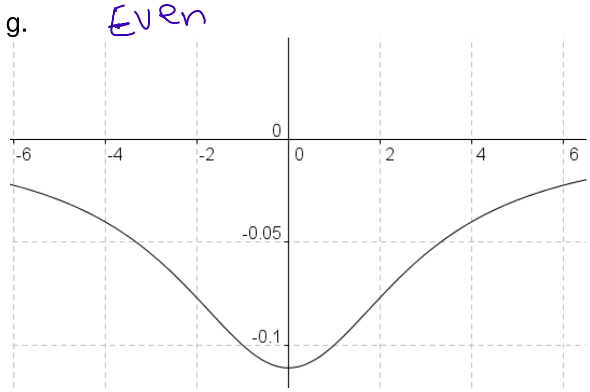
f. $y = \dfrac{2}{x^2 - 1}$

$\dfrac{2}{(-x)^2 - 1}$

$= \dfrac{2}{x^2 - 1}$

$= y$ ∴ Even

g. Even



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Journal Unit 1 # 6 and 7