Polynomial Functions Unit 3

Tentative TEST date_______________

Big idea

In this unit will you extend your knowledge from linear, quadratic and cubic functions (learned in gr9-11) to general polynomial functions. Recall that the quadratic function can be written in 3 forms – factored, expanded/standard and vertex forms. When you study polynomials you will also use all three of these forms. The vertex form will just be called the transformed form since a polynomial function may have many turning points, not just one vertex as a parabola did. You will learn that polynomial functions can be described by their end-behaviour, symmetry, number of zeros and number of turning points. You will be introduced to how to sketch the polynomial function from factored form as well as how to find the equation from a given graph. The main part of this unit will be to learn how to factor polynomials, which can be a long process involving long division or synthetic division. You will not learn how to find where exactly the turning points will occur – that is part of calculus.

<u>Corrections for the textbook answers:</u>
Sec 3.2 #12 get rid of the word range
Sec 3.3 #14 graph is wrong in answer
Sec 3.4 #6f) (-11, -3), (-4, -2), (10, 6)

Success Criteria

☐ I <u>understand the new topics</u> for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them	**Questions I had difficulty with** ask teacher before test!
	1-5	INVESTIGATIONS Polynomials, Expanded and Factored Forms Section 3.1 & 3.2 & TWO Handouts		
	6-7	Graphing and Finding Equations Section 3.3 & TWO Handouts		
	8-10	Transformed Form of Polynomial Functions Section 3.4 & Handout		
	11-12	Long Division & Synthetic Division Section 3.5 & TWO Handouts		
	13-15	Theorems and Applications THREE Handouts		
	16-18	Factoring Polynomials Section 3.6 & TWO Handouts		
	19-20	Factoring Sum & Difference of Cubes Section 3.7 & Handout		
		REVIEW		

Reflect – previous TEST mark ___________, Overall mark now___________.

What is a Polynomial Function?

1. How can you tell if the equation or graph or table that is given is of a polynomial function?

Equation: expanded form $f(x) = ax^n \pm bx^{n-1} \pm \cdots \pm cx^0$ $n \in W$ (0,1,2,3,....)

Graph: Continuous + smooth graph with infinite end behaviour.

Table: polynomial of degree "n" will have the nth constant difference

ex: linear function → polynomial function of degree "1" $y=2x^1$

x	y	1st difference
1	2	
2	4	4-2=2
3	6	6-4=2
4	8	8-6=2

Quadratic function → degree "2" $y=x^2$

x	y	1st	2nd difference
1	1		
2	4	4-1=3	
3	9	9-4=5	5-3=2
4	16	16-9=7	7-5=2

2. Identify whether the folowing are considered to be polynomial functions.

a. $y = 2x - 1$ Yes

b. $f(x) = 3x^{\frac{1}{2}} - x$ No $\frac{1}{2}$ ≠ whole number!

c. $h(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$ Not ≠ whole #. cube root

d. $y = x^3 + 2x^2 - x + 11$ Yes

e. $y = -0.2(4x-3)(x+3)$ yes $= -0.2(4x^2 + 9x - 9)$

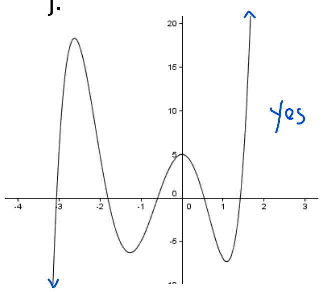
f. $y = x^{-3} + 2x^2 - x^{-1} + 11$ NO $x^{-3} = \frac{1}{x^3}$

g. $x = -6$ No because this is a vertical line ≠ function!

h. $y = 2x^0$ Yes ✱ special case!

i. $y = \frac{1}{x} = x^{-1}$ No

j.



Yes

k.

x	y
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8

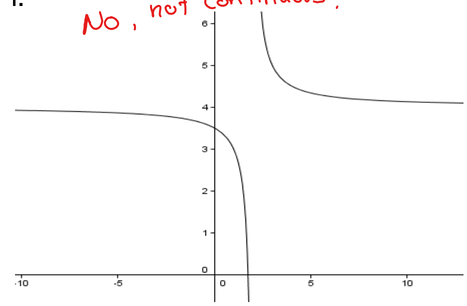
① -27-(-8) = -19; -8-(-1) = -7; -1-0 = -1; 0-1 = -1; 1-8 = -7

② -12; -6; 0; 6

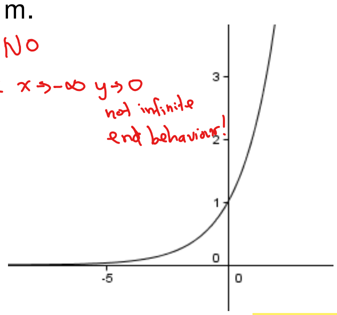
③ -6; -6; -6

∴ Polynomial of degree "3" function

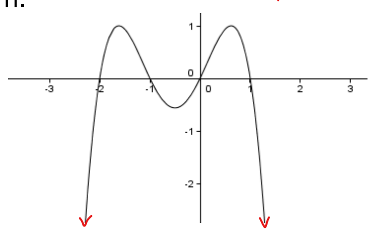
l. No, not continuous!



m. No b/c $x \to -\infty$ $y \to 0$ not infinite end behaviour!

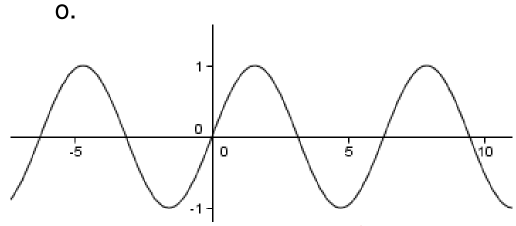


n.



Yes

o.



No, this is a sine function. No infinite end behaviour.

3. How can you find the degree and the leading coefficient of a polynomial? Explain using these examples

$y = 3x^1 - 9x^3 + 10x^6 - 1x^2 + 8x^0$ ← highest exponent of x. # infront of the term with highest exponent

Degree is 6

Leading Coefficient = 10 (L.C)

$y = -2(x-4)(3x^3 - x^2)(1 - x^4)$ ← factored form

expand roughly: $-2(x)(3x^3)(-x^4) = 6x^8$

∴ Degree is 8

L.C. is 6

Expanded form ex: $y = 3x^2 + x + 1$

① Expanded form $y = ax^n \pm a_2x^{n-1} \pm \cdots \pm a_{n-1}x^1 \pm a_nx^0$
② Factored form $y = a(\ \ )(\ \ )(\ \ )\ldots$
③ Transformed form x only appears once!

4. Identify the form of the polynomial, the degree and the leading coefficient.

a. $y=(x+1)^3(1-2x)(5x^4+x^3+1)$
Factored form
$(x^3)(-2x)(5x^4) = -10x^8$
Degree = 8
L.C = −10

b. $y=x^3-3x+7x^4-9x^5+1$
Expanded form
Degree: 5
L.C = −9

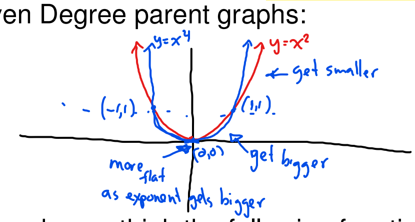
c. $y=5(1-3x)^6+8$
Transformed form
$5(-3x)^6 = 5(-3)^6x^6 = \underline{3645}x^6$
degree = 6
L.C = 3645

d. $y=-(1-4x)^3$
Transformed form
$-(-4)^3$
Degree = 3
L.C = $-(-4)^3 = 64$

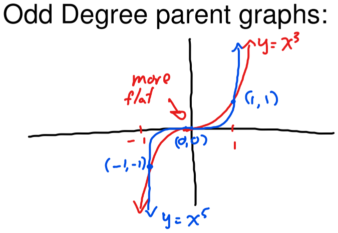
INVESTIGATE Transformed Form of Polynomials.

5. Investigate how does the degree affect the shape of the graph?

a. Even Degree parent graphs:

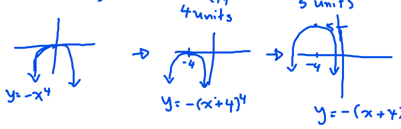


b. Odd Degree parent graphs:



6. How do you think the following functions will look?

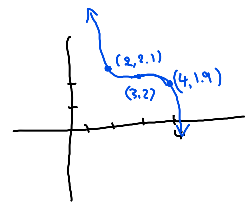
a. $y=-(x+4)^4+5$
Parent function: $y=x^4$
Reflected in x-axis
d = −4 shift left 4 units
c = 5 shift up 5 units



b. $y=-0.1(x-3)^5+2$
a = −0.1
① Reflected in x-axis
② vertical compressed by 0.1
d = 3 shift right 3 units
c = 2 shift up 2 units

Mapping Rule: $(x,y) \rightarrow (\frac{x}{k}+d, ay+c)$

$y=x^5$		$(x+3, -0.1y+2)$
(−1,−1)	→	(2, 2.1) *
(0,0)	→	(3, 2) *
(1,1)	→	(4, 1.9) *



INVESTIGATE Standard/Expanded Form of Polynomials.

7. Using technology, investigate how does having many terms affect the shape of the graph. Draw all possible twists that can occur

a. Degree 3
No turning point
2 turning points

b. Degree 4
1 turning point
3 turning points

c. Degree 5
No turning point
2 turning points
4 turning points

d. Degree 6
1 turning point
3 turning points
5 turning points

8. Investigate how does the leading coefficient affect the end behaviour of the graph?

Even function:
Positive L.C. ↑ ↗ as $x \rightarrow \pm\infty, y \rightarrow +\infty$
Negative L.C. ↙ ↘ as $x \rightarrow \pm\infty, y \rightarrow -\infty$

ODD function:
Positive L.C. as $x \rightarrow -\infty, y \rightarrow -\infty$; $x \rightarrow +\infty, y \rightarrow +\infty$
Negative L.C. as $x \rightarrow -\infty, y \rightarrow +\infty$; $x \rightarrow +\infty, y \rightarrow -\infty$

9. What conclusions can you make about the range of polynomial graphs?

Even function: $R = \{y \in \mathbb{R} \mid y \geq$ Absolute min (smallest min) [+L.C.] or $y \leq$ Absolute max (largest max) [−L.C.]$\}$
ODD function: $R: \{y \in \mathbb{R}\}$

$\text{min}_2 < \text{min}_1$, min_2 absolute min
$\text{max}_1 > \text{max}_2$ ∴ max_1 is absolute max

10. Summarize

Absolute maximum/minimum can only occur for ___EVEN___ degree polynomials.
A polynomial of EVEN degree n can have at least ___zero___, and at most ___n___ zeros.
A polynomial of EVEN degree 8 can have ___1, 3, 5, 7___ turning points
$n \rightarrow 1, 3, 5, \ldots, n-1$
A polynomial of ODD degree n can have at least ___1___, and at most ___n___ zeros.
A polynomial of ODD degree 11 can have ___0, 2, 4, 6, 8, 10,___ turning points.
n $0, 2, \ldots, n-1$