MHF4U: Functions

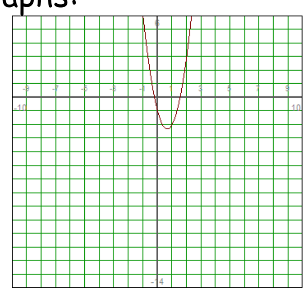
Unit 1
Activity 1
Characteristics of functions Homework - SOLUTIONS

Check your own answers now. If you experience any difficulties, email your teacher with the Unit#/Activity# and the question number. Then outline your specific problem.

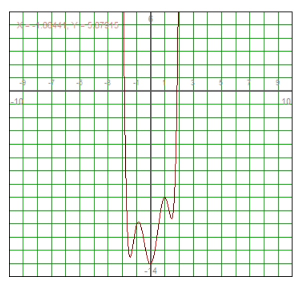
1. For each of the following functions complete the chart and sketch the function using Graph Calc.

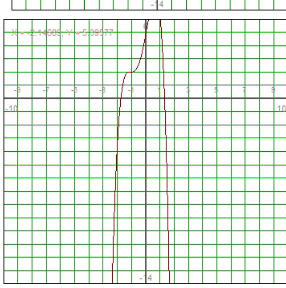
Function	degree	Sign of leading coefficient	End behaviour $x \to \infty$	End behaviour $x \to -\infty$
a) $y = 3x^2 - 4x - 1$	2	+	$y \to \infty$	$y \to \infty$
b) $y = -x^4 - x^3 + x^2 + 3$	4	-	$y \to -\infty$	$y \to -\infty$
c) $y = 2x^6 - 9x^4 + 11x^2 + x - 13$	6	+	$y \to \infty$	$y \to \infty$
d) $y = -2x^4 - 4x^3 + x^2 + 6x + 5$	4	-	$y \to -\infty$	$y \to -\infty$
e) $y = -x^3 + 8x^2 - 4x + 1$	3	-	$y \to -\infty$	$y \to \infty$

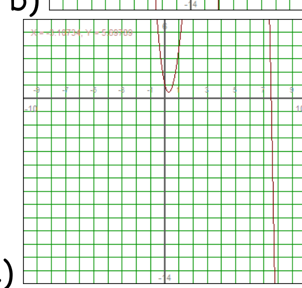
Graphs:

a) 

b) 

c) 

d) 

e) 

2. Based on the above chart, find the relationship between the end behaviour of a function and the degree. **If the degree is even, the end behaviours are the same; if the degree is odd, the end behaviours are opposite.**

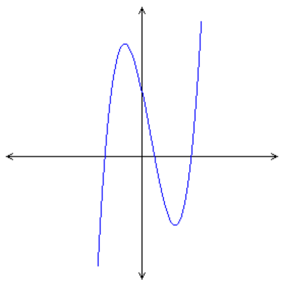
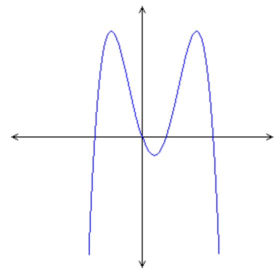
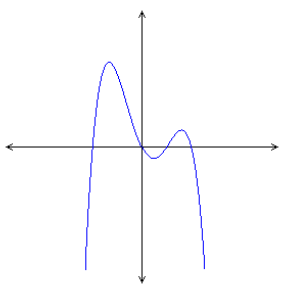
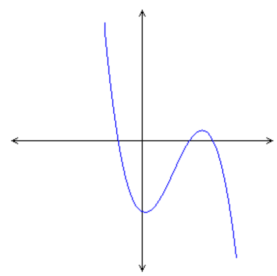
3. Using GraphCalc fill-in the following chart:

Function	degree	Number of zeroes
$y = 3x^2 - 4x - 1$	2	2
$y = -x^4 - x^3 + x^2 + 3$	4	2
$y = 2x^6 - 9x^4 + 11x^2 + x - 13$	6	2
$y = -2x^4 - 4x^3 + x^2 + 6x + 5$	4	2
$y = -x^3 + 8x^2 - 4x + 1$	3	1

4. Using Graphcalc and your results from question #3, what is the minimum and maximum number of zeros possible for each of the following polynomial functions with the following degrees:

a) 2 (min: no zeroes; max: 2 zeroes) (b) 3 (min: one zero; max: three zeroes) (c) 5 (min: one zero; max: five zeroes) (d) 6 (min: no zeroes; max: six zeroes) (e) 7 (min: one zero; max: 7 zeroes) (f) n (min: no zeroes if n is even, one zero if n is odd; max: n zeroes)

5. Identify the sign of the leading coefficient and describe the end behaviour. Using this information, decide if each function is cubic or quartic.

 Lead Coeff: +; cubic; end behaviours, see chart below.	 Lead Coeff: -; quartic; end behaviours, see chart below.
 Lead Coeff: -; quartic; end behaviours, see chart below.	 Lead Coeff: -; cubic; end behaviours, see chart below.

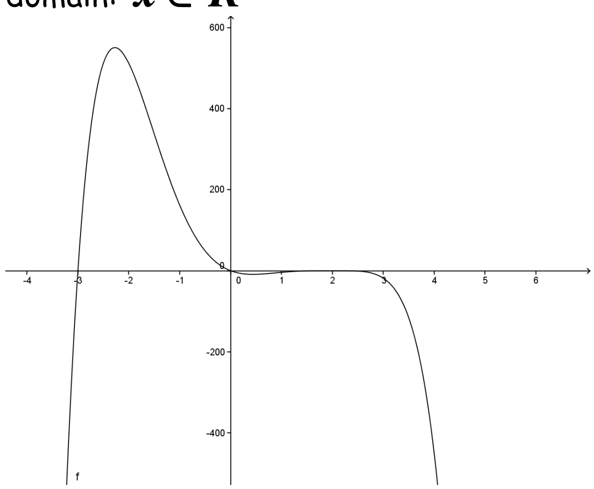
End Behaviours in General

Degree	Lead Coefficient	End behaviour $x \to \infty$	End behaviour $x \to -\infty$
Odd	+	$y \to \infty$	$y \to -\infty$
Odd	-	$y \to -\infty$	$y \to \infty$
Even	+	$y \to \infty$	$y \to \infty$
Even	-	$y \to -\infty$	$y \to -\infty$

6. Without using any graphing technology, graph the following polynomial functions after identifying the features listed:

Function: $y = -x(x-2)^4(x+3)$

Degree: 6
Sign: Negative
Quadrants: 3 to 4
end behaviours: as $x \to -\infty$,
$y \to$ __$-\infty$____
as $x \to \infty$,
$y \to$ __$-\infty$____
Roots: -3, 0, 2 (quadruple root)
Bounce at? 2
Bend at? none
Straight through at? 0 and 3
y-intercept: 0
domain: $x \in R$

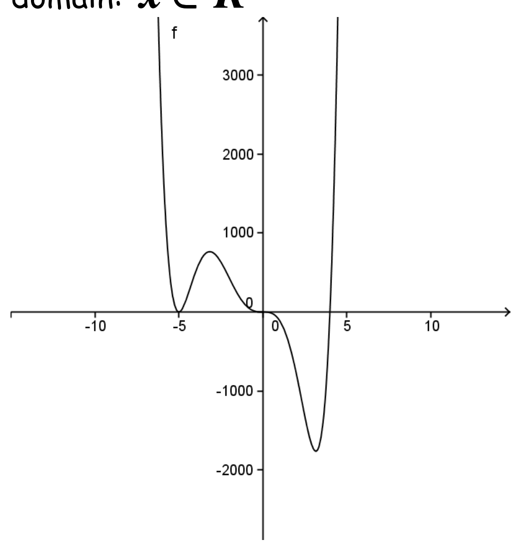


For the following you must use graphing software. (In Calculus you'll be able to determine the turning points too.)

range: $y \leq 557$
intervals of increase: $x < -2.3; 0.4 < x < 2$
intervals of decrease: $-2.3 < x < 0.4; x > 2$

Function: $y = x^3(x-4)(x+5)^2$

Degree: 6
Sign: Positive
Quadrants:2 to 1
end behaviours: as $x \to -\infty$,
$y \to$ _∞_____
as $x \to \infty$,
$y \to$ __∞____
Roots:
-5 (double root), 0 (triple root), 4
Bounce at? -5
Bend at? 0
Straight through at? 4
y-intercept: 0
domain: $x \in R$

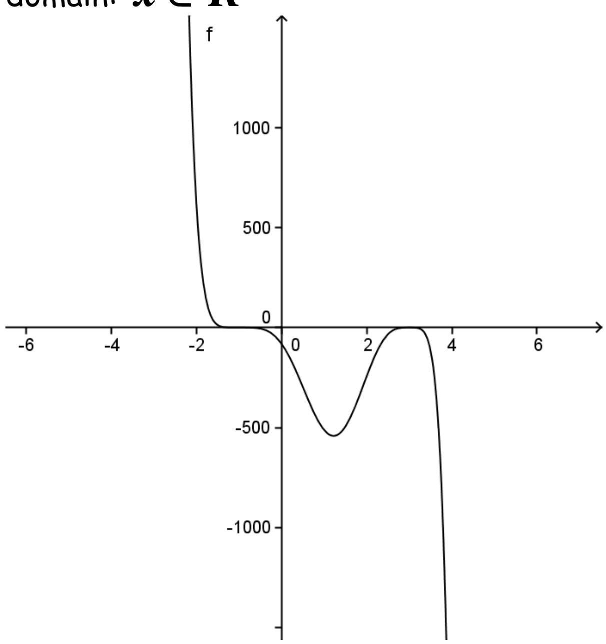


For the following you must use graphing software. (In Calculus you'll be able to determine the turning points too.)

range: $y \geq -1758$
intervals of increase: $-5 < x < -3.1; x > 3$
intervals of decrease: $x < -5; -3.1 < x < 3$

Function: $y = -(x+1)^5(x-3)^4$

Degree: 9
Sign: Negative
Quadrants: 2 to 4
end behaviours: as $x \to -\infty$,
$y \to$ __ ∞ ____
as $x \to \infty$,
$y \to$ __ $-\infty$ ____
Roots: -1, 3
Bounce at? 3
Bend at? -1
Straight through at? none
y-intercept: -81
domain: $x \in R$

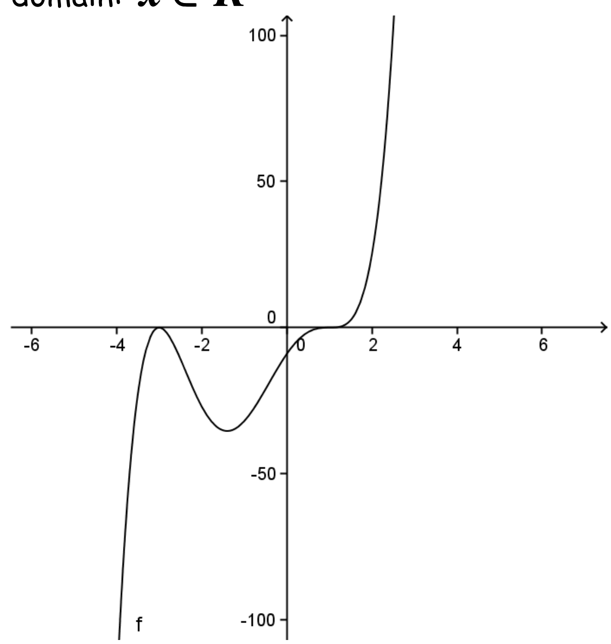


For the following you must use graphing software. (In Calculus you'll be able to determine the turning points too.)

range: $y \in R$
intervals of increase: $1.2 < x < 3$
intervals of decrease: $y < 1.2; y > 3$

Function: $y = (x+3)^2(x-1)^3$

Degree: 5
Sign: Positive
Quadrants: 3 to 1
end behaviours: as $x \to -\infty$,
$y \to$ _ $-\infty$ _____
as $x \to \infty$,
$y \to$ __ ∞ ____
Roots: -3 (double root), 1
Bounce at? -3
Bend at? 1
Straight through at? none
y-intercept: -9
domain: $x \in R$

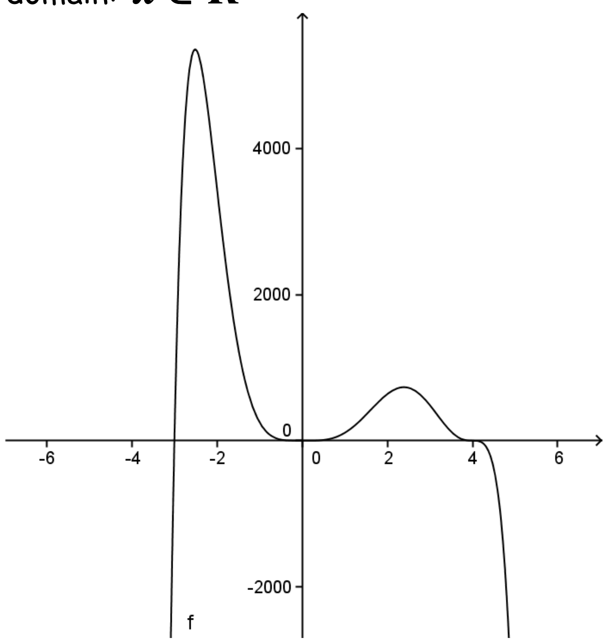


For the following you must use graphing software. (In Calculus you'll be able to determine the turning points too.)

range: $y \in R$
intervals of increase: $x < -3; x > -1.4$
intervals of decrease: $-3 < x < -1.4$

Function: $y = -x^4(x+3)(x-4)^3$

Degree: 8
Sign: Negative
Quadrants: 3 to 4
end behaviours: as $x \to -\infty$,
$y \to$ _− ∞_____
as $x \to \infty$,
$y \to$ ____− ∞__
Roots: -3, 0, 4
Bounce at? 0
Bend at? 4
Straight through at? -3
y-intercept: 0
domain: $x \in R$

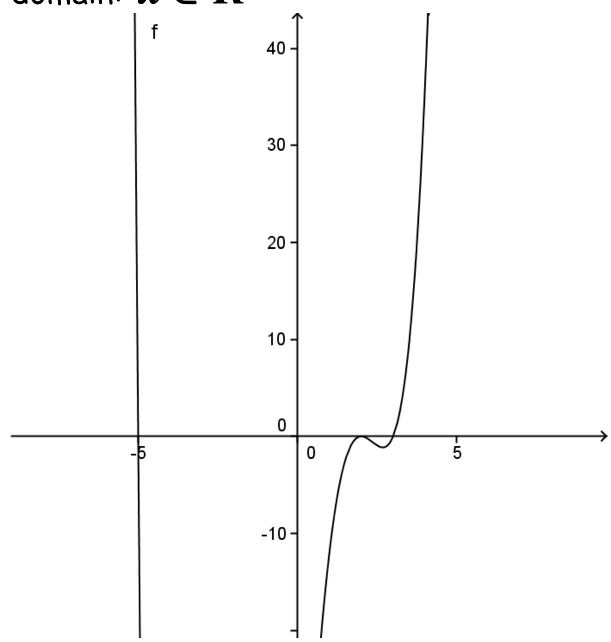


For the following you must use graphing software. (In Calculus you'll be able to determine the turning points too.)

range: $y \le 5390$
intervals of increase: $x < -2.5; 0 < x < 2.4$
intervals of decrease: $-2.5 < x < 0; x > 2.4$

Function: $y = (x-3)(x+5)(x-2)^2$

Degree: 4
Sign: Positive
Quadrants: 2 to 1
end behaviours: as $x \to -\infty$,
$y \to$ _∞_____
as $x \to \infty$,
$y \to$ _∞_____
Roots: -5, 2 (double root), 3
Bounce at? 2
Bend at? none
Straight through at? 3 and -5
y-intercept: ~~-80~~ −60
domain: $x \in R$



For the following you must use graphing software. (In Calculus you'll be able to determine the turning points too.)

range: $y \ge -300$
intervals of increase: $-3.2 < x < 2; x > 2.7$
intervals of decrease:
$x < -3.2; 2 < x < 2.7$