REVIEW - Polynomial Functions **SOLUTIONS**

Section 1: 1.1 Power Functions

1) State the degree and the leading coefficient of each polynomial

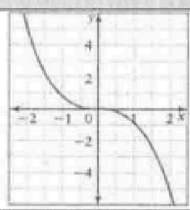
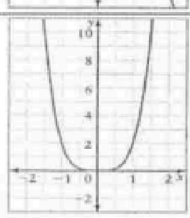
Polynomial	Degree	Leading Coefficient
$y = 2x^3 + 3x - 1$	3	2
$y = 5x - 6$	1	5
$y = x^3 - 2x^2 - 5x^4 + 3$	4	-5
$y = -3x^5 + 2x^3 - x - 1$	5	-3
$y = 21 - 2x + 4x^2 - 6x^3$	3	-6

2) Match each function to its end behavior

$y = 3x^7$ $\quad\quad$ $y = -\frac{1}{2}x^3$ $\quad\quad$ $y = 2x^4$ $\quad\quad$ $y = -0.25x^6$

End Behaviour	Functions
Q3 to Q1	$y = 3x^7$
Q2 to Q4	$y = -\frac{1}{2}x^3$
Q2 to Q1	$y = 2x^4$
Q3 to Q4	$y = -0.25x^6$

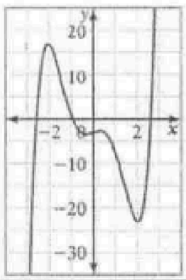
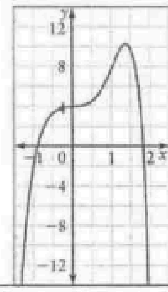
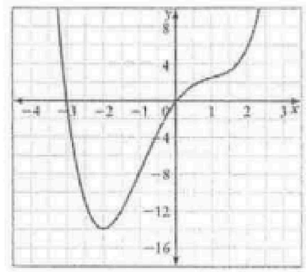
3) Complete the following table

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
	Odd	−	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$	Odd	Q2 → Q4 $x \to -\infty, y \to +\infty$ $x \to +\infty, y \to -\infty$
	Even	+	D: $(-\infty, \infty)$ R: $[0, \infty)$	Even	Q2 → Q1 $x \to \pm\infty, y \to +\infty$

Section 2: 1.2 Characteristics of Polynomial Functions

4) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

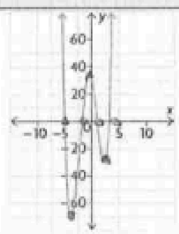
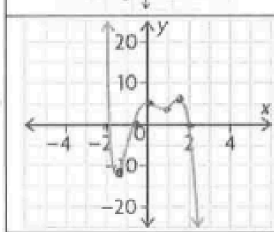
A) $g(x) = 0.5x^4 - 3x^2 + 5x$ **B)** $h(x) = x^5 - 7x^3 + 2x - 3$ **C)** $p(x) = -x^6 + 5x^3 + 4$

		
B	C	A

5) Complete the following table

Equation	Degree	Sign of Leading Coefficient	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = 6x^3 + 2x$	3	+	Q3→Q1	2, 0	3, 2, 1
$g(x) = -20x^6 - 5x^3 + x^2 - 17$	6	−	Q3→Q4	5, 3, 1	6,5,4,3,2,1,0
$p(x) = 22x^4 - 4x^3 + 3x^2 - 2x + 2$	4	+	Q2→Q1	3, 1	4,3,2,1,0
$h(x) = -x^5 + x^4 - x^3 + x^2 - x + 1$	5	−	Q2→Q4	4, 2, 0	5,4,3,2,1

6) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	+	Even	Q2→Q1	None	3	4	4
	−	Odd	Q2→Q4	None	4	3	5

7) State the degree of the polynomial function that corresponds to each constant finite difference.

a) fifth differences = -60 Degree 5

b) third differences = 42 Degree 3

9) Use finite differences to determine the degree

a)

x	y
−3	124
−2	41
−1	8
0	1
1	−4
2	−31
3	−104
4	−247

1st: -83, -33, -7, -5, -27, -73, -143

2nd: 50, 26, 2, -22, -46, -70

3rd: -24, -24, -24, -24, -24

Degree = 3

b)

x	y
−2	−229
−1	−5
0	3
1	−7
2	−53
3	−129
4	35
5	1213

1st: 224, 8, -10, -46, -76, 164, 1178

2nd: -216, -18, -36, -30, 240, 1014

3rd: 198, -18, 6, 270, 774

4th: -216, 24, 264, 504

5th: 240, 240, 240

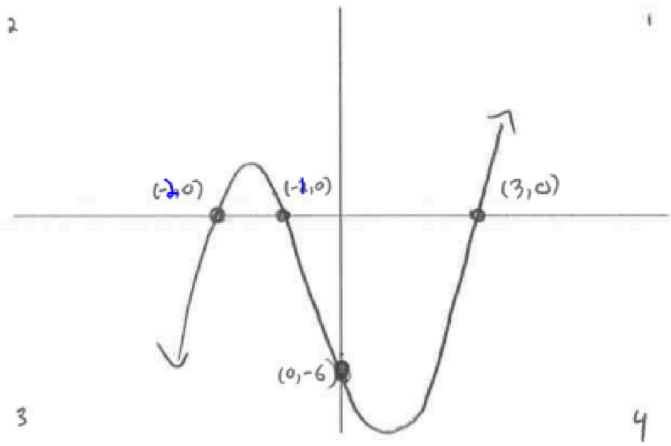
Degree = 5

Section 3: 1.3 Factored Form Polynomial Functions

10) For each function, complete the chart and sketch a possible graph of the function labelling key points.

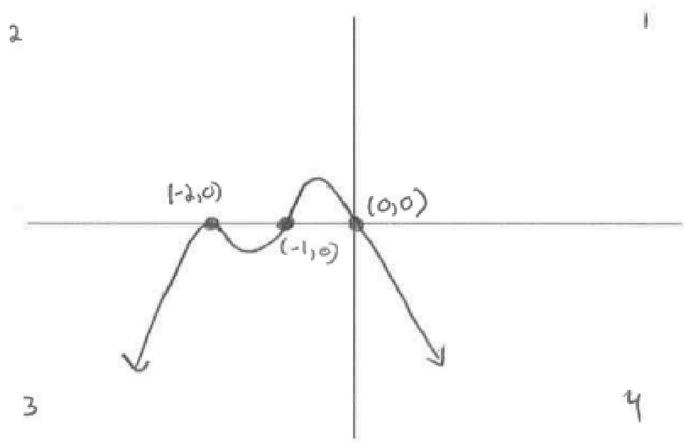
a) $f(x) = (x+1)(x-3)(x+2)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)$ $= x^3$ Degree 3	$(1)(1)(1)$ $= 1$	Q3 → Q1 $x \to -\infty, y \to -\infty$ $x \to +\infty, y \to +\infty$	$(-1, 0)$ $(3, 0)$ $(-2, 0)$	$f(0) = (0+1)(0-3)(0+2)$ $= (1)(-3)(2)$ $= -6$



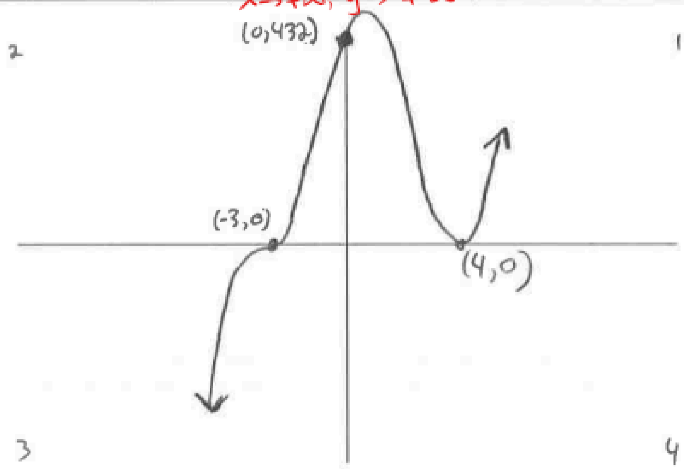
b) $g(x) = -x(x+1)(x+2)^2$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x^2)$ $= x^4$ Degree 4	$-1(1)(1)^2$ $= -1$	Q3 → Q4 $x \to \pm\infty, y \to -\infty$	$(0, 0)$ $(-1, 0)$ $(-2, 0)$ order 2	$g(0) = -(0)(0+1)(0+2)^2$ $= -(0)(1)(4)$ $= 0$



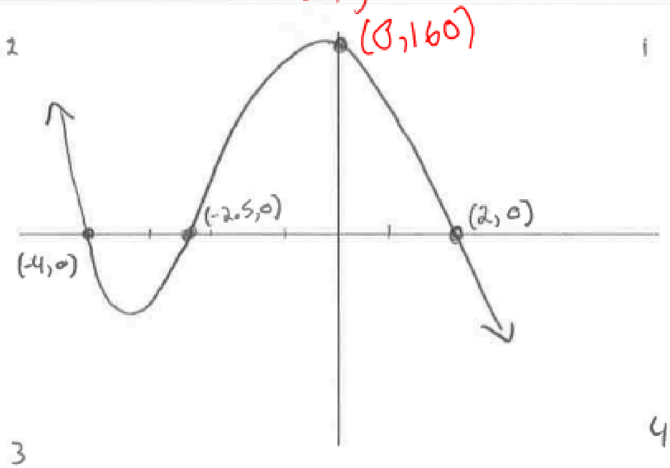
c) $h(x) = (x-4)^2(x+3)^3$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x^2)(x^3)$ $= x^5$ Degree 5	$(1)^2(1)^3$ $= 1$	Q3 → Q1 $x \to -\infty, y \to -\infty$ $x \to +\infty, y \to +\infty$	(4,0) order 2 (-3,0) order 3	$h(0) = (0-4)^2(0+3)^3$ $= (16)(27)$ $= 432$



d) $p(x) = -4(2x+5)(x-2)(x+4)$

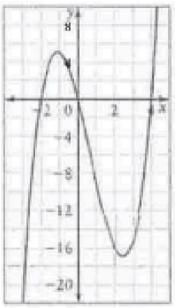
Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)$ $= x^3$ Degree 3	$-4(2)(1)(1)$ $= -8$	Q2 → Q4 $x \to -\infty, y \to +\infty$ $x \to +\infty, y \to -\infty$	$(-5/2, 0)$ (2,0) (-4,0)	$p(0) = -4[2(0)+5](0-2)(0+4)$ $= -4(5)(-2)(4)$ $= 160$



11) For each graph, state...

i) the least possible degree and the sign of the leading coefficient
ii) the x-intercepts (specify order of zero) and the factors of the function
iii) the intervals where the function is positive/negative

a)



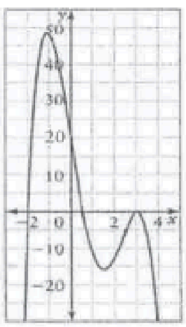
i) degree: 3
leading coefficient: POSITIVE

ii) x-intercepts: $-2, 0, 4$
factors: $(x+2)$, x, $(x-4)$

iii)

Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 4)$	$(4, \infty)$
Sign	−	+	−	+

b)



i) degree: 4
leading coefficient: NEGATIVE

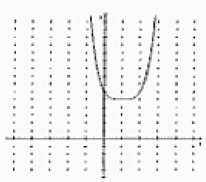
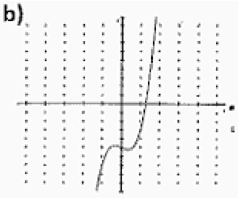
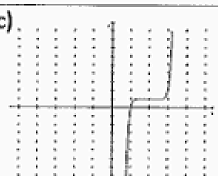
ii) x-intercepts: -2, $\frac{1}{2}$, 3 (order 2)
factors: $(x+2)$, $(2x-1)$, $(x-3)^2$

iii)

Interval	$(-\infty, -2)$	$(-2, 0.5)$	$(0.5, 3)$	$(3, \infty)$
Sign	−	+	−	−

Section 5: 1.5 Symmetry

15) Circle all that apply for each function

a) 	Neither No symmetry Even function Odd function	d) $f(x) = 3x^6 + 2x^2 - 5$x^0	No symmetry Even function Odd function
b) 	Neither No symmetry Even function Odd function	e) $f(x) = x^3 - 4x^2 + 1$	No symmetry Neither Even function Odd function
c) 	Neither No symmetry Even function Odd function	f) $f(x) = x^4 + 5x$	No symmetry Neither Even function Odd function

16) Consider the polynomial function $f(x) = -3x^4 + 6x^2 - 10$

a) Show algebraically whether *f* is even, odd or neither.

$f(-x) = -3(-x)^4 + 6(-x)^2 - 10$

$f(-x) = -3(-1)^4(x)^4 + 6(-1)^2(x)^2 - 10$

$f(-x) = -3x^4 + 6x^2 - 10$

$\therefore f(-x) = f(x)$

Since $f(-x) = f(x)$, it is an even function.

b) For what finite difference will *f* give a constant value

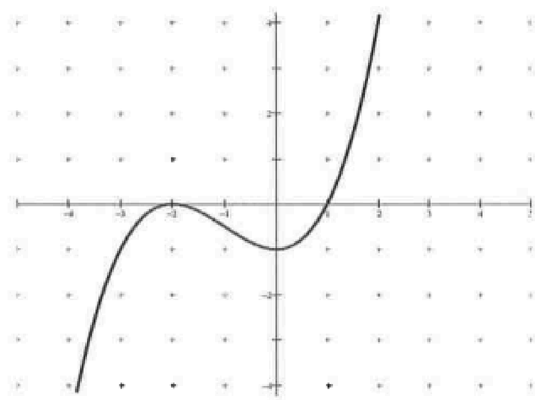
It is degree 4, so the 4th differences will be constant.

c) What are the maximum and minimum number of zeros the above polynomial could have?

Min Zeros = 0
Max Zeros = 4

Possible zeros for degree 4 are 4, 3, 2, 1, or 0.

17) Use the given graph to state:



a) x-intercepts -2 (order 2), and 1

b) number of turning points 2

c) least possible degree 3

d) any symmetry present; even or odd function?

NEITHER

Not an even or odd function.

e) the intervals where $f(x) < 0$

$f(x) < 0$ when: $x < -2$ or $-2 < x < 1$

$f(x) < 0$ when $x \in (-\infty, -2) \cup (-2, 1)$