Graphing and Finding Equations

1. Summarize the steps of sketching polynomials given expanded form. Explain what details you still don't know about the graph.

2. Sketch each of the following

a. $f(x) = 4x^2 - 5x^5 + x - 1$

Degree $= 5 \Rightarrow$ odd

L.C. $= -5$ ↑ ↓

b. $f(x) = 3 - 7x + 8x^4 - x^3$

c. $f(x) = 11x - 9x^4 + 10x^7$

d. $f(x) = -6x^6 + 8x^2 - x^3 + 9$

3. Summarize the steps of sketching polynomials given factored form. Explain what details you still don't know about the graph.

4. Sketch each of the following

a. $y = x^1(x+3)^2(2-2x)^1$

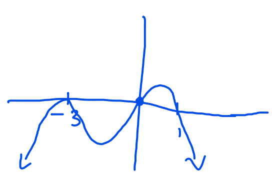
Degree $= 1+2+1 = 4$ Even

L.C. $= (1)(1)^2(-2) = -2$ ↙ ↘

zeros: 0, $\underbrace{-3 \text{ (order 2)}}_{\text{bounce}}$, 1

y-int: $y = 0$

t.p. =



b. $y = (x+3)^2(2-4x)^2$

c. $y = (2-x)^3$

d. positive leading coefficient,
zeros at -3, -1 (order 5), 2, 4 (order 4)

Sketch each of the following

e. $y = -x(x+2)^3(1-x)^4$

f. $y = -3(2x+4)^2(x-1)$

g. $y = (6-2x)^4$

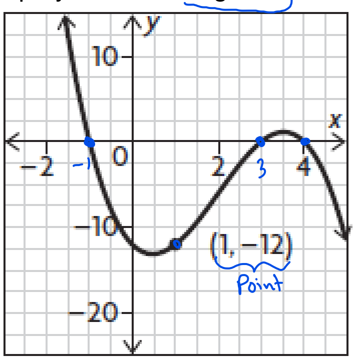
h. $y = (2x+3)^3(5-x)$

factored form

5. Find the equations for the given graphs

$y = a(\ )^n(\ )$

a. polynomial of degree 3 with the following graph



* Cannot use the zero as a point!

Zeroes: $-1, 3, 4$

$y = a(x+1)^1(x-3)^1(x-4)^1$

Sub point $(1, -12)$ — x, y

$-12 = a(1+1)(1-3)(1-4)$

$-12 = (2)(-2)(-3)a$

$-12 = +12a$

$-1 = a$

$\therefore y = -(x+1)(x-3)(x-4)$

b. The function is cubic with zeros at 4(order 2) and -1 and with a y-intercept of 10.

exponent 2

$x = 0$

$y = a(x-4)^2(x+1)$ point $(0, 10)$ — x, y

$10 = a(0-4)^2(0+1)$

$10 = 16a$

$\frac{10}{16} = a$

$\div 2$, $\div 2$: $\frac{5}{8} = a$

reduced fraction

$\therefore y = \frac{5}{8}(x-4)^2(x+1)$

6. The function $f(x) = kx^3 + 20x^2 - 8x - 96$ has a zero when $x = -3$. Determine the value of k.

$0 = k(-3)^3 + 20(-3)^2 - 8(-3) - 96$ $f(x) = y = 0$

$k = 4$

Transformed Form of Polynomials

$y = a[k(x-d)]^n + c$

* factor k so nothing in front of x!

1. Write out the transformed form of a polynomial of degree n. Summarize what type of transformations the letters control and recap the steps of transforming a function.

a → reflection in x-axis when $a<0$
→ vertical stretch ($|a|>1$) or compress ($0<|a|<1$) by "$|a|$"

k → reflection in y-axis when $k<0$
→ horizontal stretch ($0<|k|<1$) or compress ($|k|>1$) by "$\frac{1}{|k|}$"

d → Horizontal shift left ($d<0$) or right ($d>0$)

c → vertical shift down ($c<0$) or up ($c>0$)

$\boxed{y = a[k(x-d)]^n + c}$

Parent function → transformed function

x	$f(x)$

→

$\frac{x}{k}+d$	$ay+c$

2. Sketch the following

* Rewrite the equation to $y = a[k(x-d)]^n + c$

a. $f(x) = -2(\frac{1}{4}x - 1)^3 - 8$

$= -2\left[\frac{1}{4}(x-4)\right]^3 - 8$

$a = -2$
$k = \frac{1}{4}$
$d = 4$
$c = -8$

$\frac{x}{1/4} = 4x + 4$

$(\frac{x}{k}+d,\ ay+c)$

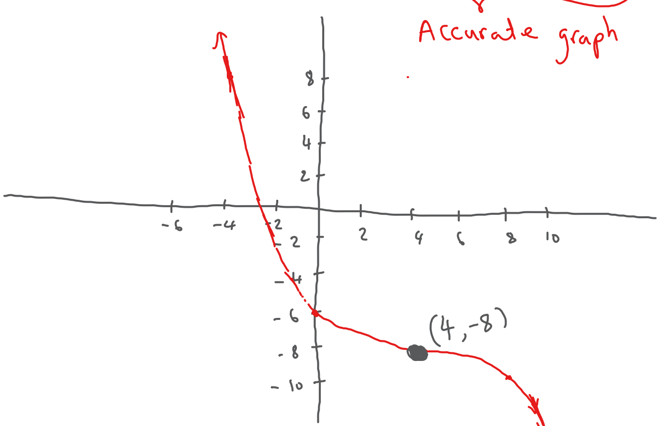
Parent function: $y = x^3$ → transformed func.

key points

x	$y = x^3$
-2	-8
-1	-1
0	0
1	1
2	8

x: $4x+4$	y: $-2y-8$
$4(-2)+4 = -4$	$-2(-8)-8 = 8$
$4(-1)+4 = 0$	$-2(-1)-8 = -6$
$4(0)+4 = 4$	$-2(0)-8 = -8$
$4(1)+4 = 8$	$-2(1)-8 = -10$
$4(2)+4 = 12$	$-2(8)-8 = -24$

Accurate graph



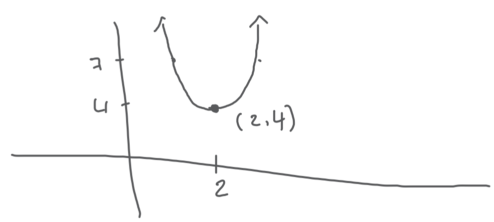
b. $f(x) = 3(10 - 5x)^4 + 4$

$= 3[-5(x-2)]^4 + 4$

Parent function → Transformed func.

x	$y = x^4$
-1	1
0	0
1	1

$\frac{x}{-5}+2$	$3y+4$
$\frac{11}{5} = 2.2$	7
2	4
$\frac{9}{5} = 1.8$	7



3. Explain why it is important to follow the steps outlined above instead of just taking the x values below and just plugging them into the equation. Use question a. as an example in your explanation.

x	y
-2	
-1	
0	
1	
2	

$y = a[k(x-d)]^n + c$ $n \leftarrow$ degree

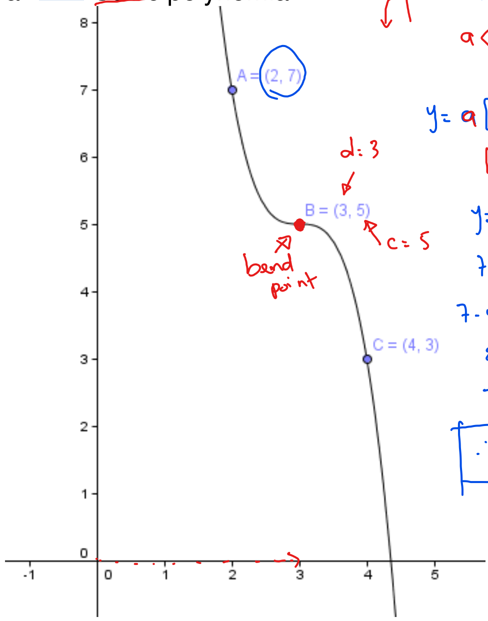
let $k=1$ then solve for a quintic $\Rightarrow n=5$

4. Find the equations for the following graphs. Be careful not all pictures represent a transformed form of a parent!!!

a. Cubic polynomial $n=3$

x^3 Parent function

$a<0$



$y = a[k(x-3)]^3 + 5$

let $k = 1$

$y = a[1(x-3)]^3 + 5$

$7 = a(2-3)^3 + 5$

$7 - 5 = -a$

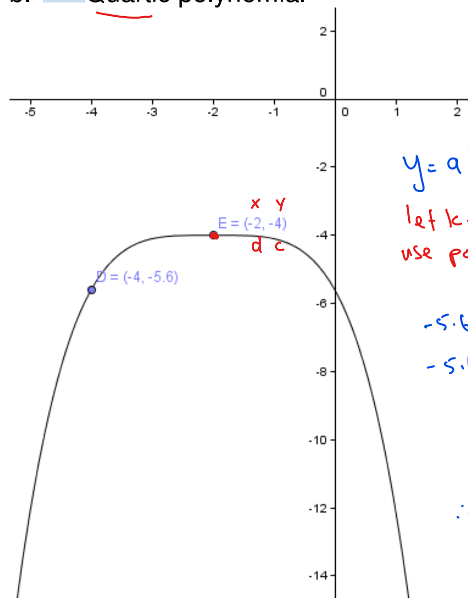
$2 = -a$

$-2 = a$

$\boxed{\therefore y = -2(x-3)^3 + 5}$

b. Quartic polynomial $n=4$

Parent



$y = a[k(x+2)]^4 - 4$

let $k = 1$

use point $(-4, -5.6)$

$-5.6 = a(-4+2)^4 - 4$

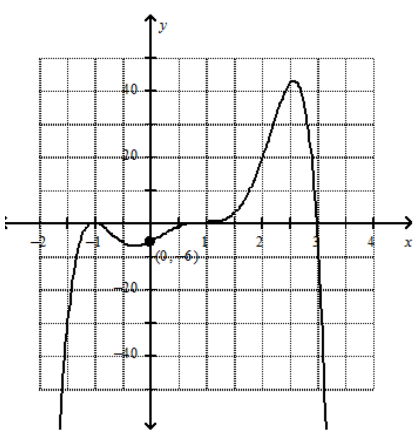
$-5.6 + 4 = 16a$

$-1.6 = 16a$

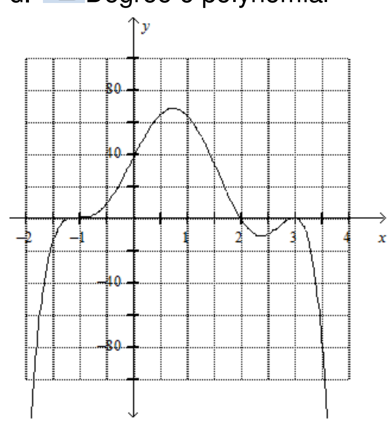
$-0.1 = a$

$\therefore y = -0.1(x+2)^4 - 4$

c. Degree 6 polynomial   factored form



d. Degree 6 polynomial   factored form.



Set $y=0$, solve for x.

5. Determine the zeros of the following polynomials

even root!
$\pm\sqrt[2]{\ }$
$\pm\sqrt[4]{\ }$
$\pm\sqrt[6]{\ }$

a. $y = 3(4x-8)^4 - 243$

$0 = 3(4x-8)^4 - 243$

$243 = 3(4x-8)^4$

$\frac{243}{3} = (4x-8)^4$

$81 = (4x-8)^4$

$\pm\sqrt[4]{81} = 4x-8$

$\pm 3 + 8 = 4x$

$\frac{\pm 3 + 8}{4} = x$

$x_1 = \frac{3+8}{4} = \frac{11}{4}$ or $x_2 = \frac{-3+8}{4} = \frac{5}{4}$

b. $y = 2(2x-10)^3 + 128$

$0 = 2(2x-10)^3 + 128$

$\frac{-128}{2} = (2x-10)^3$

$\sqrt[3]{-64} = 2x-10$

$-4 = 2x-10$

$\frac{6}{2} = x$

$\boxed{3 = x}$

odd root
$\sqrt[3]{\ }$
$\sqrt[5]{\ }$
$\sqrt[7]{\ }$
No $\pm$

c. Why can't the above method be used on the following $y = -5x^4 - 4x^2 + 1$?

d. What method do you know of that could help you find the roots of $y = -5x^4 - 4x^2 + 1$?

e. Would the method you used for d. work on $y = -5x^4 - 4x^3 + 1$?