Long Division and Synthetic Division of Polynomials

1. Remind yourself how to do long division with numbers, since the same approach will be used for polynomial division.

a. Identify where the following terms would be located on the long division table: Quotient, Divisor, Dividend, Remainder, then find the quotient and remainder.

$\frac{25697}{4}$

Quotient: 6424

Divisor → $4\overline{)25697}$ ← Dividend

24
1697
− 16
097
− 8
17
− 16
1 ← Remainder

b. State the formula that relates all of these terms together and modify it to the one that has dividend – which will always be the function, f(x) – isolated.

① $\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$

② * Dividend = (Quotient)(Divisor) + Remainder *

2. Provide an explanation of how to do long division.

① Fill in any missing terms with zeros.
② Divide only 1st terms each time.
③ Multiply the result of step ② with whole divisor
④ Subtract
⑤ Repeat after carry down the next terms.

3. Provide an explanation of how to do synthetic division.

① find the "root" of divisor
② use only coefficient of dividend, for missing terms use 0 coefficient
③ Carry down 1st term
④ Multiply by the "root" of divisor
⑤ Add
⑥ Repeat.

4. Divide using long division, write down the result in TWO different ways

a. $(2x^4 - 5x^2 + 3) \div (x + 4)$

$2x^4 + 0x^3 - 5x^2 + 0x + 3$

$\frac{?}{(x)} = 2x^4$ $(2x^3)(4) = 8x^3$ $(-)(x) = -8x^3$ $(27x)(x) = 27x^2$

$2x^3 - 8x^2 + 27x - 108$ ← Quotient

Divisor → $x+4 \overline{)2x^4 + 0x^3 - 5x^2 + 0x + 3}$ ← Dividend

$-\ 2x^4 + 8x^3$
$0\ \ -8x^3 - 5x^2 + 0x + 3$
$-\ \ -8x^3 - 32x^2$
$0\ + 27x^2 + 0x + 3$
$-\ \ 27x^2 + 108x$
$0\ \ -108x + 3$
$-\ \ -108x - 432$
$0 + 435$ ← Remainder

$\therefore \frac{2x^4 - 5x^2 + 3}{x+4} = 2x^3 - 8x^2 + 27x - 108 + \frac{435}{x+4}$

OR

$2x^4 - 5x^2 + 3 = (2x^3 - 8x^2 + 27x - 108)(x+4) + 435$

b. $(12x^4 - 6x^3 - x + 81) \div (3x^2 - 9)$

Dividend, Divisor

$4x^2 - 2x + 12$

$3x^2 + 0x - 9 \overline{)12x^4 - 6x^3 + 0x^2 - x + 81}$

$-\ 12x^4 + 0x^3 - 36x^2$
$\phi\ \ -6x^3 + 36x^2 - x + 81$
$-\ \ -6x^3 + 0x^2 + 18x$
$\phi\ \ 36x^2 - 19x + 81$
$-\ \ 36x^2 + 0x - 108$
$\phi\ \ -19x + 189$

$\therefore \frac{2x^4 - 6x^3 - x + 81}{3x^2 - 9} = 4x^2 - 2x + 12 + \frac{-19x + 189}{3x^2 - 9}$

or

$2x^4 - 6x^3 - x + 81 = (4x^2 - 2x + 12)(3x^2 - 9) + (-19x + 189)$

→ only work for linear divisor! degree = 1

5. Divide using synthetic division, don't forget to write down the result.

a. $\underbrace{(4x^5-2x^4+3x-9)}_{\text{Dividend}} \div \underbrace{(x-2)}_{\text{Divisor}}$ ↖ Root = +2

$4x^5 - 2x^4 + 0x^3 + 0x^2 + 3x - 9$

coefficient of Dividend

Root of Divisor → 2

	4	-2	0	0	3	-9
2		8	12	24	48	102
	4	6	12	24	51	$\boxed{93}$
	x^4	x^3	x^2	x^1	x^0	

Quotient | Remainder

$\therefore 4x^5-2x^4+3x-9 = (4x^4+6x^3+12x^2+24x+51)(x-2)+\boxed{93}$

↖ always a #

b. $(2x^3+5x^2-4x-5)\div\underbrace{(2x+1)}_{(x+\frac{1}{2})}$ → Root = $-\frac{1}{2}$

$2x^3+5x^2-4x-5 = (2x^2+4x-6)(x+\frac{1}{2}) - 2$

6. What are the disadvantage(s) of long division?

→ too much writing

7. What are the disadvantages of synthetic division?

→ Only work for Linear divisors

8. Practice both methods

a. $(x^3-8x^2-4x-7)\div(x^2+1)$

$$
\begin{array}{r|l}
 & \quad x - 8 \\
\hline
x^2+0x+1 & x^3 - 8x^2 - 4x - 7 \\
 & -(x^3 + 0x^2 + x) \\
\hline
 & -8x^2 - 5x - 7 \\
 & -(-8x^2 + 0x - 8) \\
\hline
 & -5x + 1
\end{array}
$$

$\therefore x^3-8x^2-4x-7 = (x-8)(x^2+1)-5x+1$

b. $(12x^4-20x^3-4x+24)\div\underbrace{(4x-8)}_{x-2}$

	12	-20	0	-4	24
2		24	8	16	24
	12	4	8	12	$\boxed{48}$ R

$\therefore 12x^4-20x^3-4x+24 = (12x^3+4x^2+8x+12)(4x-8)+48$

c. $(3x^3-x^2+10x)\div(x-2)$ Root = 2

	3	-1	10	0
2		6	10	40
	3	5	20	$\boxed{140}$ R

$\therefore 3x^3-x^2+10x = (3x^2+5x+20)(x-2)+40$

d. $(10x^3-14x+5)\div(x+1)$ → Root = -1

	10	0	-14	5
1		10	10	-4
	10	10	-4	$\boxed{1}$ R

$\therefore 10x^3-14x+5 = (10x^2+10x-4)(x+1)+1$