Theorems and Applications

Remainder Theorem:
When a polynomial $f(x)$ is divided by a linear divisor $(x-a)$, then the remainder is equal to $f(a)$.
(If the divisor is not linear then the remainder might not be just one constant number but a polynomial – and this theorem will not work)

Proof:
divident
$\boxed{f(x)=(quotient)(divisor)+remainder}$ ☆

divisor is $x-a$

$f(x)=(quotient)(x-a)+remainder$

sub in the zero of the divisor $x=a$

$f(a)=(quotient)(0)+remainder$

$f(a)=remainder$

1. Find the remainder (without doing the division) of $f(x)=x^3-12x^2-42$ when it is divided by $x-3$

$R=f(3)=3^3-12(3)^2-42=-123$

check with synthetic:

3	1	−12	0	−42
	↓	3	−27	−81
	1	−9	−27	$\boxed{-123}$ = R

2. Determine the remainder, r, to make this multiplication statement true
$(x^2-4)(3x^2+1)+r=3x^4-10x^2+5$
quot. divisor dividend
thrm. won't work here.

FOIL, isolate r

$\cancel{3x^4}+x^2-12x^2-4+r=\cancel{3x^4}-10x^2+5$
$-3x^4$

$r=x^2+9$

3. If the dividend is $x^4+x^3-7x^2+6x-2$, quotient is $x^3-3x^2+5x-14$ and remainder is 54, find the divisor by using comparing coefficients method.

given 3 of 4 things in the formula ☆

dividend = (quot.)(divisor) + rem.

$x^4+x^3-7x^2+6x-2=(x^3-3x^2+5x-14)(ax+b)+54$

linear? quad? ?

∴ Divisor is
$ax+b$
$=1x+4$

Compare x^4 term
LS = RS
$1x^4=ax^4$
$1=a$

Compare x^3
LS = RS
$x^3=bx^3+-3ax^3$
$1x^3=(b-3a)x^3$
$1=b-3a$
$1=b-3(1)$
$4=b$

faster to do 1st + Last

compare constant term
$-2=54-14b$
$-56=-14b$
$4=b$

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TEAM.

4. Find the remainder (without doing the division) of $f(x)=2x^5+4x^4-5x^3-6x^2+2x-4$ when it is divided by $x+3$

$\therefore R=-91$ ✓

Team

5. If the divisor is x^2+5x+8, the dividend is $x^4-5x^3+3x^2-7x+11$. Find the remainder.

$\therefore R=-152x-349$ ✓ ✓

6. If the dividend is $-2x^4-5x^3+2x^2+3x-1$, quotient is $-2x^3-7x^2-5x-2$ and remainder is – 3, find the divisor by using comparing coefficients method.

$-2x^4-5x^3+2x^2+3x-1=(-2x^3-7x^2-5x-2)(ax+b)-3$

compare x^4

$-2x^4=-2ax^4$

$\frac{-2x^4}{-2x^4}$ $\frac{}{-2x^4}$ isolate

$1=a$ ✓

compare constant term

$-1=-2b-3$

$+3$

$2=-2b$

$-1=b$ ✓

$\therefore$ Divisor is $(x-1)$ ✓

7. Remind yourself what is the definition of the word "factor"

↳ divides evenly

or remainder is zero.

by remainder theorem $f(a)=0$ ✱ see this in factor th.