Quadratic $f(x) = (x-1)(x+2)$ $x = 1$ or -2 $f(x) = (x-1)(2x+3)$ $x = 1$ or $x = -\frac{3}{2}$ ← fractional root

Rational Root Theorem:

If a polynomial $y = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ has any rational (integer or fractional) zeros then the zeros are from the list of all combinations of $\pm\frac{p}{q}$, where p is an integer factor of a_0 and q is an integer factor of a_n.

(Keep in mind that zeros can also be irrational – these can only be found using technology. In university you'll study how the technology is programmed to find the zeros using different approximation methods)

$\pm$ # $\frac{q}{p}$ q p

8. Find the list of possible zeros of

$f(x) = 3x^3 - 5x^2 + 5x - 2$

$q = 3$ $(1, 3)$ $p = -2$ $(1, 2)$

$\pm\frac{p}{q}$: $\pm\frac{2}{3}$, $\pm\frac{2}{1} = \pm 2$, $\pm\frac{1}{3}$, $\pm\frac{1}{1} = \pm 1$

9. Find the list of possible zeros of

$f(x) = 2x^3 + 3x^2 - 18x + 8$

$q = 2 \Rightarrow (1, 2)$ $p = 8 \Rightarrow (1, 2, 4, 8)$

$\pm\frac{p}{q}$: $\pm\frac{1}{1}$, $\pm\frac{1}{2}$, $\pm\frac{4}{1}$, $\pm\frac{8}{1}$, $\pm\frac{2}{1}$, ~~...~~

± 1, $\pm\frac{1}{2}$, ± 4, ± 8, ± 2

Factor Theorem:

The polynomial $f(x)$ has a factor $(x-k)$ if and only if (this means the implication goes both ways) $f(k) = 0$

Proof - "if" implication:

if $x - k$ is a factor of $f(x)$
then it divides into $f(x)$ evenly
This means the remainder is zero.
By Remainder theorem $f(k) = remainder$
Therefore $f(k) = 0$, since remainder is zero.

Proof - "only if" implication:

if $f(k) = 0$
then by Remainder theorem $remainder = 0$
but that implies the divisor $x - k$ goes into $f(x)$ evenly
which means the divisor $x - k$ is a factor.

10. Which of the numbers from the list you've made for $f(x) = 3x^3 - 5x^2 + 5x - 2$ above, are actual zeros?

Possible Zeros are: $\pm 1, \pm 2, \pm\frac{2}{3}, \pm\frac{1}{3}$

Sub $x=1$, $f(1) = 3(1)^3 - 5(1)^2 + 5(1) - 2 = 1$

$f(-1) = -15$

Kyo $f(2) = 12$

Cici $f(-2) = -56$

Mia $f(\frac{1}{3}) = -\frac{7}{9}$

Kevin $f(-\frac{1}{3}) = -\frac{13}{3}$

Jane $f(\frac{2}{3}) = 0$

$f(-\frac{2}{3}) = -\frac{76}{9}$

$\therefore$ The actual zero is $x = \frac{2}{3}$

$f(x) = (x - \frac{2}{3})(ax^2 + bx + c)$

11. Determine if the binomial $2x - 3$ is a factor of

$f(x) = 2x^5 + 3x^3 + 6$

→ zero

$2x - 3 = 0$
$x = \frac{3}{2}$

$f(\frac{3}{2}) \overset{?}{=} 0$

$f(\frac{3}{2}) = 2(\frac{3}{2})^5 + 3(\frac{3}{2})^3 + 6$

$\neq 0$

$\therefore$ $2x - 3$ is not a factor of $f(x)$.

12. Which of the numbers from the list you've made for $f(x) = 2x^3 + 3x^2 - 18x + 8$ above, are actual zeros? Use your answers to factor the polynomial completely.

$f(-1) =$

$f(1) =$

$f(-2) =$

$f(2) = 0$

$f(-4) = 0$

$f(4) =$

$f(-8) =$

$f(8) =$

$f(-\frac{1}{2}) =$

$f(\frac{1}{2}) = 0$

$\therefore f(x) = a(x-2)(x+4)(x - \frac{1}{2})$

$f(x) = 2(x-2)(x+4)(x - \frac{1}{2})$

13. Determine if the binomial $x + 4$ is a factor of

$x = -4$

$f(x) = x^3 + 5x^2 + 2x - 8$

$f(-4) \overset{?}{=} 0$

$f(-4) = (-4)^3 + 5(-4)^2 + 2(-4) - 8$

$= 0$

$\therefore$ $x + 4$ is a factor of $f(x)$

$f(x) = (x+4)(ax^2 + bx + c)$

Homework:
1. Journal Unit 3 # 6b
2. Review Note pages 13 – 15.