Prime number
↳ number can only divides itself and 1.

$3x^2 + 2x - 1$ Factor → $(3x-1)(x+1)$

Factoring Polynomials

1. Find the prime factorization of 324. How does this help find the factors of polynomial such as in question a. below?

$324 = (2)^2 (3)^4$

$324 \to 2$, $\searrow 162 \to 2$, $\searrow 81 \to 3$, $\searrow 27 \to 3$, $\searrow 9 \to 3$, $\to 3$

2. How do you think you can factor a cubic polynomial by just finding one zero? What about a quartic?

① use trial and error to find which $\pm\frac{p}{q}$ combination makes the function $= 0$.

② Use synthetic division to find the quotient.

③ Record result: $f(x) = (\text{quotient})(\text{divisor}) + R$ * $R = 0$.

④ Factor the quotient more
↳ if quadratic then you do it the same as what you learn in grade 10 math
↳ if cubic, repeats step 1 to 3.

3. Factor completely by first finding one zero

a. $f(x) = x^5 - 45x^3 + 324x$ long way: ① Common factor

Common factor x

$= x(x^4 - 45x^2 + 324)$ $q = 1$, $p = 324$

$\pm\frac{p}{q}: \frac{\pm 324}{1}, \pm\frac{1}{1}, \pm\frac{9}{1}, \pm\frac{6}{1}, \pm\frac{4}{1}, \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{2}{1}$

$f(6) = 0$ $\therefore (x-6)$ is a factor

Use Synthetic Division:

$$\begin{array}{r|rrrrr} 6 & 1 & 0 & -45 & 0 & 324 \\ & & 6 & 36 & -54 & -324 \\ \hline & 1 & 6 & -9 & -54 & \boxed{0} \leftarrow R \end{array}$$

quotient: $x^3 + 6x^2 - 9x - 54$

$\therefore f(x) = x(x-6)(x^3 + 6x^2 - 9x - 54) + 0$ factor

Repeat $\pm\frac{p}{q}$....

$f(3) = 0$ $\therefore (x-3)$ is a factor

$$\begin{array}{r|rrrr} 3 & 1 & 6 & -9 & -54 \\ & & 3 & 27 & 54 \\ \hline & 1 & 9 & 18 & \boxed{0} \leftarrow R \end{array}$$

quotient

$\therefore f(x) = x(x-6)(x-3)(x^2 + 9x + 18)$

$f(x) = x(x-6)(x-3)(x+6)(x+3)$

b. $f(x) = x^5 - 45x^3 + 324x$ shortcut :
only works if variable pattern is like criss cross factoring

$f(x) = x(x^4 - 45x^2 + 324)$ let $a = x^2$

$= x(a^2 - 45a + 324)$

$= x(a-9)(a-36)$ * Difference of squares. $a^2 - b^2 = (a-b)(a+b)$

$= x(x^2-9)(x^2-36)$ ← Stop Here?

$= x(x-3)(x+3)(x-6)(x+6)$

* ① Look for common factor *
② Look for Special Cases
* ③ Factor using Long way. *

c. $12x^3 + 8x^2 - 3x - 2$

$= (2x+1)(2x-1)(3x+2)$

or $= \left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right)\left(x+\frac{2}{3}\right)$

d. $3x^3 + 8x^2 - 21x + 6$

$= (x-1.4)(x+4.4)\left(x-\frac{1}{3}\right)$

When completing practice questions, you may use the following online calculator to help you check your answers, or to use to find zeros of polynomials for which you get stuck on or the ones that have only irrational solutions – textbook sometimes does give questions that are unfactorable over rationals.
http://www.mathportal.org/calculators/polynomials-solvers/polynomial-roots-calculator.php

Half $\frac{1}{2}$ third $\frac{1}{3}$ quarter $\frac{1}{4}$ Tripple $\times 3$ quadruple $\times 4$ fifth $\frac{1}{5}$

Thinking

4. When ax^3+3x^2-2x-b is divided by $x-1$, the remainder is 1. When it is divided by $x-2$, the remainder is 40. Find $a+b$

$x-1=0$, $x=1$, $f(1)=1$

$x-2=0$, $x=2$, $f(2)=40$

Remainder Theorem:

$x=1$

$f(1)=1$

$a(1)^3+3(1)^2-2(1)-b=1$

$a-b+1=1$

$a-b=0$ ①

$a=b$

$f(2)=40$

$a(2)^3+3(2)^2-2(2)-b=40$

$8a+12-4-b=40$

$8a-b=32$ ②

Solve by elimination:

②-①: $8a-b=32$

$-\ (a-b=0)$

$7a=32$

$a=\frac{32}{7}$ $\therefore b=\frac{32}{7}$

Sub $a=\frac{32}{7}$ in ① solve for b.

$\frac{32}{7}-b=0$

$\frac{32}{7}=b$

$\boxed{\therefore a+b=\frac{32}{7}+\frac{32}{7}=\frac{64}{7}}$

5. For the function $f(x)=x^3-5x^2+kx-16$, the remainder from $f(x)\div(x+1)$ is twice the remainder from $f(x)\div(x-1)$. Determine the value of k.

$R_1=2R_2$

$f(-1)=2(f(+1))$

$(-1)^3-5(-1)^2+k(-1)-16=2\left[(1)^3-5(1)^2+k(1)-16\right]$

$k=6$

Difference of Squares exponent 2
$a^2 - b^2 = (a-b)(a+b)$

cubes ⇒ exponent 3

Factoring a Sum and a Difference of Cubes

1. Develop the SUM of CUBES formula by factoring the following polynomial $x^3 + k^3$:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

2. The DIFFERENCE of CUBES formula is similarly derived. The formula is:

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

* 2 terms !

① Common Factor
② Special Cases
↳ difference of squares
↳ difference of cubes
↳ Sum of cubes.

3. Factor the following by using the formulas developed above

a.

$\frac{8}{343}x^3 - \frac{1}{216}$

$8 = 2^3$ $343 = 7^3$ $1 = 1^3$ $216 = 6^3$

$a = \frac{2}{7}x$ $b = \frac{1}{6}$

$= \left(\frac{2}{7}x\right)^3 - \left(\frac{1}{6}\right)^3$

$= \left(\frac{2}{7}x - \frac{1}{6}\right)\left[\left(\frac{2}{7}x\right)^2 + \left(\frac{2}{7}x\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)^2\right]$

$= \left(\frac{2}{7}x - \frac{1}{6}\right)\left(\frac{4}{49}x^2 + \frac{1}{21}x + \frac{1}{36}\right)$

b.

$-54x^4 - 250x$ Sum of cubes

$= -2x(27x^3 + 125)$ $a = 3x$ $b = 5$

$= -2x(3x+5)((3x)^2 - (3x)(5) + 5^2)$

$= -2x(3x+5)(9x^2 - 15x + 25)$

c.

$(x-5)^3 - (5x-1)^3$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$a = x-5 \qquad a-b = (x-5)-(5x-1) = -4x-4$

$b = 5x-1$

$a^2 = (x-5)^2 = x^2-10x+25$

$b^2 = (5x-1)^2 = 25x^2-10x+1$

$ab = (x-5)(5x-1) = 5x^2-26x+5$

$= (-4x-4)[(x^2-10x+25)+(5x^2-26x+5)+(25x^2-10x+1)]$

$= (-4x-4)(31x^2-46x+31)$ ← Stop Here -0.25

$= -4(x+1)(31x^2-46x+31)$

Homework:

1. Journal Unit 3 #6c, 7, 8
2. Review package for test Mon, Jan. 26.
(You will have 30mins to ask question & review before test)

Perfect Square:

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2-2ab+b^2$

d.

$(2a+b)^3 + (2a-b)^3$

$= [(2a+b)+(2a-b)][(2a+b)^2-(2a+b)(2a-b)+(2a-b)^2]$

$= 4a[(4a^2+4ab+b^2)-(4a^2-b^2)+(4a^2-4ab+b^2)]$

$= 4a(\cancel{4a^2}+\cancel{4ab}+b^2\cancel{-4a^2}+b^2+4a^2\cancel{-4ab}+b^2)$

$= 4a(4a^2+3b^2)$