Introduction to Calculus Unit - Notes

Tentative TEST date Monday - Mar. 9

Big idea/Learning Goals

Calculus is an entire branch of mathematics. Calculus is built on two major complementary ideas.

Find derivatives.

High school

$y = x^3 + 2x + 1$

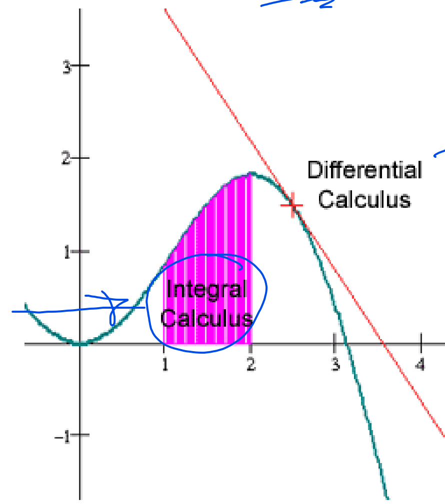
$y' = 3x^2 + 2$

Part 2

University

ex: $\int (3x^2+2)\,dx$

$y = x^2 + 2x + C$



The first is differential calculus, which is concerned with the instantaneous rate of change. This can be illustrated by the slope of a tangent to a function's graph. The second is integral calculus, which studies the areas under a curve. These two processes act inversely to each other. The development of the mathematical methods of calculus has been credited to two great mathematicians; Sir Isaac Newton (1642 –1727) and Gottfried Wilhelm von Leibniz (1646 –1716). Although others as far back as 200B.C. had been working on solutions to these types of problems, Newton and Leibniz developed the process of differentiation and integration. Calculus allows you to find optimal solutions to mathematical expressions and is used in medicine, engineering, economics, computer science, business, physical sciences, statistics, and many more areas.

Corrections for the textbook answers:
1.2 #20 500

Limit

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them
Tues.	2-4	Limit of a Function 1.4 textbook.	
Wed.	5-7	Properties of Limits 1.5	
Thu.	8-10	Continuity 1.6	
Fri.	11-13	Slopes of Secants and Tangents with Limits 1.2 & 1.3	
		Review	

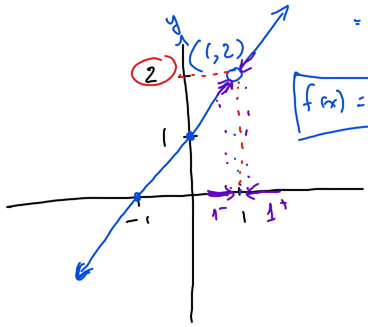
Reflect – previous TEST mark ___________, Overall mark now___________.

Limit of a Function

1. Sometimes you can't work something out directly ... but you can see what it should be as you get closer and closer!

   Why can't you find out the output of the function $f(x)=\frac{x^2-1}{x-1}$ at x=1? $f(1)=?$ $x \neq 1$

2. Sketch the function $f(x)=\frac{x^2-1}{x-1}$, $x \neq 1$
   $=\frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$ hole@ $x=1$, $y=2$
   $\boxed{f(x)=x+1}$ Zeros: $x+1=0$, $x=-1$
   y-int: $f(0)=1$
   line, slope $=1$
   y-int: 1



3. What is the output approaching as $x \to 1$? y →

   $x \to 1$, $y \to 2$

   using limit notation.

   $$\lim_{x\to 1}\left(\frac{x^2-1}{x-1}\right)=2$$

4. The notation $\boxed{\lim_{x\to a} f(x)=L}$ is read as:

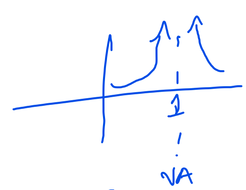
   limit of the function f(x) as x approaching a is equal to L.

5. $\boxed{\lim_{x\to a} f(x)=L}$ means that it **EXISTS** if:

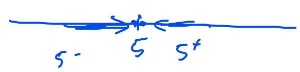
   ① $\lim_{x\to a^-} f(x)$ Exists (Left side limit)
   ② $\lim_{x\to a^+} f(x)$ Exists (Right side limit)
   ③ $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

6. Limit will not exist if there is:
   jump in the graph | one sided graph $y=\sqrt{x}$ | approaching ∞ because of a VA

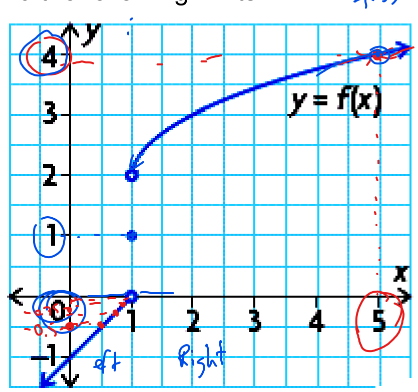
$\lim_{x\to 0^-} f(x) = DNE$



$\lim_{x\to 1^-} f(x) = \infty$
$\lim_{x\to 1^+} f(x) = \infty$



7. Find the following limits. f(5)



$x \to 4.99999$ a. $\lim_{x\to 5^-} f(x) = 4$ left

$x \to 5.00001$ b. $\lim_{x\to 5^+} f(x) = 4$ Right

e. $\lim_{x\to 5} f(x) = 4$

c. $\lim_{x\to 1^-} f(x) = 0$ left side

(does not exist)

d. $\lim_{x\to 1^+} f(x) = 2$ right side

f. $\lim_{x\to 1} f(x) = DNE$
because
$\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

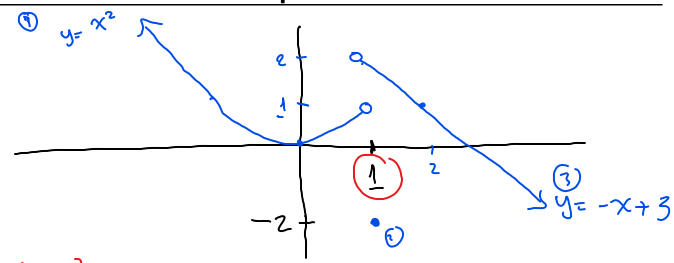
g) $f(1) = 1$

Practice:
Pg. 38 #6 & 7.

8. Determine if the limit exists both algebraically and graphically

Algebraic *informal way by trying #s from either side*	**Graphical**
a. $f(x)=\sqrt{4-x}$ limit at $x=4$	

Algebraic *informal way by trying #s on either side*	**Graphical**
b. $f(x)=\dfrac{1}{1-x}$ limit at $x=1$	

Algebraic *formal way by subbing in*	**Graphical**
c. $f(x)=\begin{cases} x^2 & \text{if } x<1 \\ -2 & \text{if } x=1 \\ -x+3 & \text{if } x>1 \end{cases}$ limit at $x=1$	

Piecewise function

left → $f(1)=1^2$ → hole. y-int → 0

slope = -1

y=

→ $f(1)=-1+3=2$ → hole

$\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^-} x^2$

$= 1^2$

$= 1$

$\lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+} (-x+3)$

$= -(1)+3$

$= 2$

$f(1)=-2$

$\therefore \lim\limits_{x\to1} f(x) = DNE$

because $\lim\limits_{x\to1^-} f(x) = 1 \neq 2 = \lim\limits_{x\to1^+} f(x)$

$\lim\limits_{x\to1^-} f(x) \neq \lim\limits_{x\to1^+} f(x)$

$\therefore \lim\limits_{x\to1} f(x) = DNE$

ALGEBRA

a) $f(x)=\sqrt{4-x}$ limit $x=4$

Right side: $\lim\limits_{x\to4^+} f(x) = \sqrt{4-4.0001}$

Sub $x=4.0001$

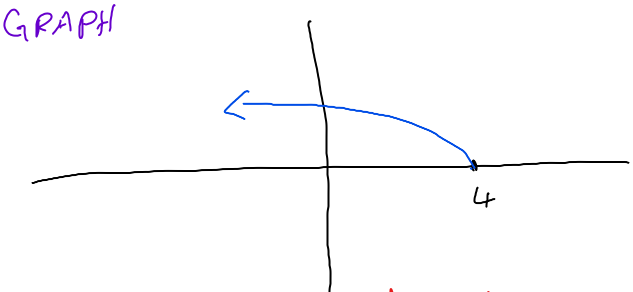
$=\sqrt{-0.0001}$

$\therefore$ DNE because cannot $\sqrt{-}$

$\therefore \lim\limits_{x\to4} f(x) = DNE$

GRAPH

$D=\{x\in\mathbb{R} \mid x\le 4\}$



Right side limit does not exist

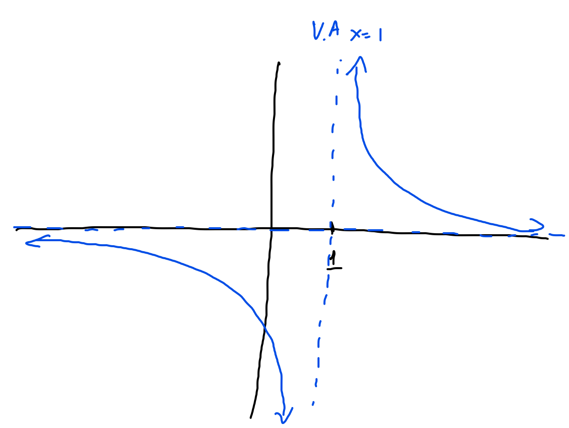
$\therefore \lim\limits_{x\to4} f(x) = DNE$

b) $f(x)=\dfrac{1}{1-x}$ limit at $x=1$ V.A.

V.A: $1-x=0$

$x=1$

H.A $y=0$



$\lim\limits_{x\to1^-} f(x) = \dfrac{1}{1-0.9999} = \dfrac{1}{0.0001} = 10000 = +\infty$

Sub $x=0.9999$

$\lim\limits_{x\to1^+} f(x) = \dfrac{1}{1-1.0001} = \dfrac{1}{-0.0001} = -10000 = -\infty$

Sub $x=1.0001$

Since $\lim\limits_{x\to1^-} f(x) \neq \lim\limits_{x\to1^+} f(x)$

$\therefore \lim\limits_{x\to1} f(x) = DNE$

Practice: pg. 39 # 1ab, 10

9. Discuss proper way of recording solutions when subbing in the value limit approaches

- Write equal sign before limit (not after)
- drop limit when you substitute only.
- $\frac{\#}{0}$ = DNE
- $\frac{0}{0}$ indeterminate form.
- 0.000001 = 0
- ±1000000 = ±∞

$= \lim_{x\to 3} \frac{x^2-9}{x-3}$

$= \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}$

$= \lim_{x\to 3} (x+3)$

sub $x=3$

$= 3+3$

$= 6$

Not $\lim_{x\to 3} = f(x)$

$= \frac{(x+3)(x-3)}{x-3}$

$= x+3$

$= 3+3$

$= 6$

X

Homework: ① Journal Unit 5 #1
② Textbook Practice Pg. 38 # 6, 7, 10
Pg. 39 # 11.ab

Determine if the limit exists both algebraically and graphically

Algebraic *formal way by subbing in*	**Graphical**
d. $f(x)=\frac{4-x^2}{x-2}$ limit at $x=2$	

Algebraic *informal way*	**Graphical**
e. $f(x)=\frac{1}{x}$ limit at $x\to\infty$	

Algebraic	**Graphical**
f. $f(x)=\frac{1}{x+2}$ limit at $x=-3$	