

Name: _____

Date: _____

MCV4U Properties of Limits Quiz

1. [5 marks] Explain why the **limit does not exist** for the following: *- show that it fails one of the following.*



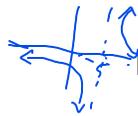
a. $\lim_{x \rightarrow 3} \sqrt{x-3}$

sub $x = 2.9999$

$$\begin{aligned} \lim_{x \rightarrow 3^-} \sqrt{x-3} &= \sqrt{2.9999-3} \\ &= \sqrt{-0.0001} \\ &= \text{DNE} \end{aligned}$$

① fail

$\therefore \lim_{x \rightarrow 3} \sqrt{x-3} = \text{DNE}$



b. $\lim_{x \rightarrow 5} \frac{1}{x-5}$

$$\begin{aligned} \lim_{x \rightarrow 5^-} \frac{1}{x-5} &= \frac{1}{4.999-5} = -\infty \quad \text{① pass} \\ \lim_{x \rightarrow 5^+} \frac{1}{x-5} &= \frac{1}{5.0001-5} = +\infty \quad \text{② pass} \end{aligned}$$

since $\lim_{x \rightarrow 5^-} f(x) \neq \lim_{x \rightarrow 5^+} f(x)$ ③ fail

$\therefore \lim_{x \rightarrow 5} f(x) = \text{DNE}$ $x^2 + x - 12 = (x-3)(x+4)$

2. [15 marks] Evaluate the following limits or justify why it doesn't exist.

a. $\lim_{x \rightarrow 8} \sqrt[3]{5x^2 - 18x - 8}$ sub $x = 8$

$$\begin{aligned} &= \sqrt[3]{5(8)^2 - 18(8) - 8} \\ &\approx 5.52 \end{aligned}$$

b. $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{2x^2 - 5x - 3}$ factor

$$\begin{aligned} &= \lim_{x \rightarrow 3} \frac{(x-3)(x+4)}{(2x+1)(x-3)} \\ &= \lim_{x \rightarrow 3} \frac{x+4}{2x+1} \quad \text{sub } x=3 \\ &= \frac{3+4}{2(3)+1} \\ &= \frac{7}{7} \\ &= 1 \end{aligned}$$

c. $\lim_{x \rightarrow -3} \frac{(1+x)^2 - 4}{x+3}$ Perfect square $(a+b)^2 = a^2 + 2ab + b^2$
 sub $x = -3 \Rightarrow \frac{0}{0}$
 expand & simplify

$$\begin{aligned} &= \lim_{x \rightarrow -3} \frac{x^2 + 2x + 1 - 4}{x+3} \\ &= \lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{x+3} \quad \text{factor} \\ &= \lim_{x \rightarrow -3} \frac{(x+3)(x-1)}{x+3} \\ &= \lim_{x \rightarrow -3} x-1 \quad \text{sub } x = -3 \\ &= -3 - 1 \\ &= -4 \end{aligned}$$

d. $\lim_{x \rightarrow \infty} \frac{(6x^2 - 5x + 2) \div x^2}{(-7x^2 + 3x) \div x^2}$ sub $x = \infty$

$$\begin{aligned} &= \lim_{x \rightarrow \infty} \frac{6 - \frac{5}{x} + \frac{2}{x^2}}{-7 + \frac{3}{x}} \\ &= \frac{6 - \frac{5}{\infty} + \frac{2}{\infty}}{-7 + \frac{3}{\infty}} \\ &= -\frac{6}{7} \end{aligned}$$

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e. $\lim_{x \rightarrow -6} \frac{2x+12}{|x+6|}$ ← absolute function
 $|x+6| = \begin{cases} -(x+6), & x < -6 \text{ left} \\ x+6, & x > -6 \text{ right} \end{cases}$

$$\begin{aligned} \lim_{x \rightarrow -6^-} \frac{2x+12}{-(x+6)} &\xrightarrow{\text{sub } x = -6} \frac{0}{0} \text{ factor} \\ &= \lim_{x \rightarrow -6^-} \frac{2(x+6)}{-(x+6)} \\ &= \lim_{x \rightarrow -6^-} -2 \\ &= -2 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow -6^+} \frac{2x+12}{x+6} \\ &= \lim_{x \rightarrow -6^+} \frac{2(x+6)}{x+6} \\ &= \lim_{x \rightarrow -6^+} 2 \\ &= 2 \end{aligned}$$

Since $\lim_{x \rightarrow -6^-} f(x) \neq \lim_{x \rightarrow -6^+} f(x)$

$\therefore \lim_{x \rightarrow -6} f(x) = \text{DNE}$

g. $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2}$ LCD

$$\begin{aligned} &= \lim_{x \rightarrow 2} \frac{\frac{2-x}{2x}}{x-2} \\ &= \lim_{x \rightarrow 2} \frac{2-x}{2x} \times \frac{1}{x-2} \\ &= \lim_{x \rightarrow 2} \frac{-(x-2)}{2x} \cdot \frac{1}{(x-2)} \\ &= \lim_{x \rightarrow 2} -\frac{1}{2x} \quad \text{Sub } x=2 \\ &= -\frac{1}{(2)(2)} \\ &= -\frac{1}{4} \end{aligned}$$

Rationalize numerator

f. $\lim_{x \rightarrow 0} \frac{\sqrt{5+x} - \sqrt{5+x}}{x}$ ← difference of squares
 $(a-b)(a+b) = a^2 - b^2$
 $\xrightarrow{\text{sub } x=0 \Rightarrow \frac{0}{0}}$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{(5+x) - (5+x)}{x(\sqrt{5+x} + \sqrt{5+x})} \\ &= \lim_{x \rightarrow 0} \frac{0}{x(\sqrt{5+x} + \sqrt{5+x})} \\ &= 0 \end{aligned}$$

h. $\lim_{x \rightarrow 0} \frac{(x+125)^{\frac{1}{3}} - 5}{x}$

$$\begin{aligned} &= \lim_{u \rightarrow 5} \frac{(u^{\frac{1}{3}})^{\frac{1}{3}} - 5}{u^3 - 125} \\ &= \lim_{u \rightarrow 5} \frac{(u-5)}{(u-5)(u^2 + 5u + 25)} \\ &= \lim_{u \rightarrow 5} \frac{1}{u^2 + 5u + 25} \\ &= \frac{1}{5^2 + 5(5) + 25} \\ &= \frac{1}{75} \end{aligned}$$

change variable

let $u^3 = x + 125$
 $x = u^3 - 125$
 when $x = 0$
 $u^3 = 0 + 125$
 $u = \sqrt[3]{125} = 5$

difference of cubes
 $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
 $(a-b)^3$
 $= (a-b)(a-b)(a-b)$