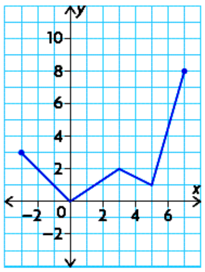
Continuity

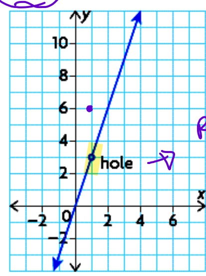
DISCONTINUOUS

A. Continuous for all values of the domain
No hole, no V.A. no jump. $\{x \in R\}$



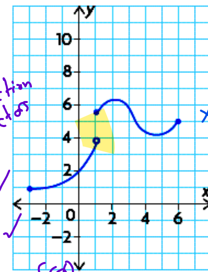
× $f(1) = DNE$
✓ $\lim_{x\to 1} f(x) = 3$

B. Discontinuous at $x = 1$
① (point discontinuity)



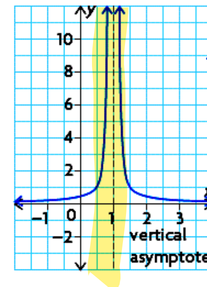
Rational function
Cancel the factors
$f(1) = 6$ ✓
$\lim_{x\to 1} f(x) = 3$ ✓
$f(1) \neq \lim_{x\to 1} f(x)$

C. Discontinuous at $x = 1$
② (jump discontinuity)



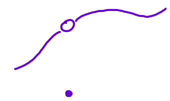
✓ $f(1) = 5.5$
× $\lim_{x\to 1} f(x) = DNE$
piecewise function

D. Discontinuous at $x = 1$
③ (infinite discontinuity)



× $f(1) = DNE$
$\lim_{x\to 1} f(x) = +\infty$
V.A.

1. State the 3 conditions you must meet for the function to be continuous.
   ① $f(a)$ Exist
   ② $\lim_{x\to a} f(x)$ Exist
   ③ $\lim_{x\to a} f(x) = f(a)$

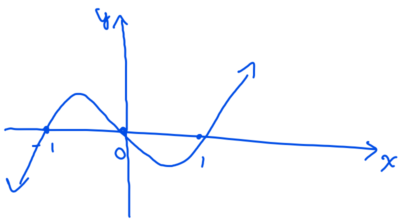


2. Determine if the function is continuous, if not, identify where the discontinuity is. Also show a graphical representation.

a. $f(x) = x^3 - x$

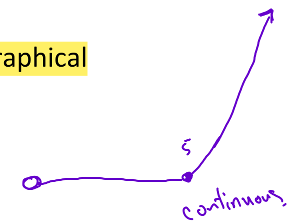
$y = x^3$ $\{x \in R\}$
$y = x$ $\{x \in R\}$
$\{x \in R\}$ $\{x \in R\}$

Polynomial function,
This function is continuous everywhere.
odd function : L.C. +
Zeros: $0 = x(x^2 - 1)$
$x = 0$ $x = \pm 1$



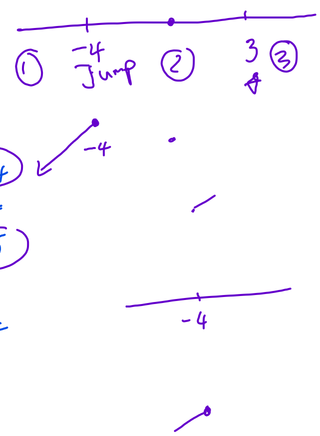
Piecewise Function.

b. $g(x) = \begin{cases} x & \text{if } x \le -4 \\ 5 & \text{if } -4 < x \le 3 \\ x^2 - 4 & \text{if } x > 3 \end{cases}$



Check at $x = -4$
① $g(-4) = -4$ ✓
② $\lim_{x\to -4} g(x) = DNE$. ×
because
$\lim_{x\to -4^-} g(x) = \lim_{x\to -4} x = -4$
$\neq$
$\lim_{x\to -4^+} g(x) = \lim_{x\to -4} 5 = 5$

∴ Jump discontinuous at $x = -4$



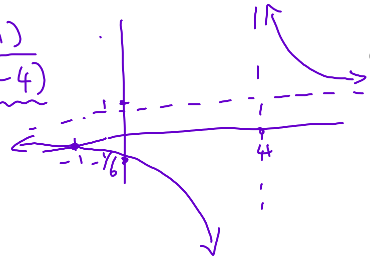
Check at $x = 3$:
① $g(3) = 5$ ✓
② $\lim_{x\to 3} g(x) = 5$ ✓
$\lim_{x\to 3^-} g(x) = \lim_{x\to 3} 5 = 5$
$\lim_{x\to 3^+} g(x) = \lim_{x\to 3} x^2 - 4 = 3^2 - 4 = 5$
③ $g(3) = \lim_{x\to 3} g(x) = 5$ ✓

∴ Continuous at $x = 3$

Determine if the function is continuous, if not, identify where the discontinuity is. Also show a graphical representation.

Rational function.

c. $h(x)=\dfrac{2x+2}{3x-12} = \dfrac{2(x+1)}{3(x-4)}$



V.A. $x-4=0 \Rightarrow x=4$

H.A. $y=\frac{2}{3}$

y-int: $-\frac{1}{6}$

x-int: $x+1=0 \Rightarrow x=-1$

$h(4)=$ DNE ! + V.A $x=4$

$\therefore$ Infinite discontinuity at $x=4$

d. $i(x)=\begin{cases} \dfrac{x^2-9}{x+3}, & \text{if } x<0 \\ -3, & \text{if } x\geq 0 \end{cases}$

has a hole at $x=-3$ b/c $\dfrac{x^2-9}{x+3} = \dfrac{\cancel{(x+3)}(x-3)}{\cancel{x+3}}$

Cancel factor $\Rightarrow$ hole

$\boxed{\therefore \text{Point discontinuity at } x=-3}$

Check at $x=0$:

1. $i(0) = -3$
2. $\lim_{x\to 0} i(x) = ?$

$\lim_{x\to 0^-} i(x) = \lim_{x\to 0} \dfrac{x^2-9}{x+3} = \lim_{x\to 0} \dfrac{\cancel{(x+3)}(x-3)}{\cancel{x+3}}$

$= \lim_{x\to 0} x-3 = 0-3 = -3$

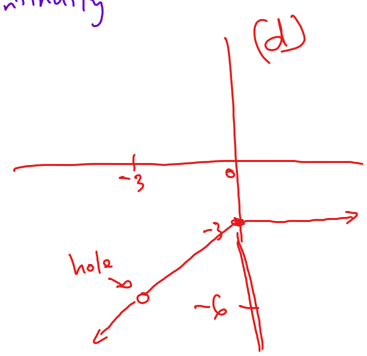
$\lim_{x\to 0^+} i(x) = \lim_{x\to 0} -3 = -3$

$\therefore \lim_{x\to 0} i(x) = -3$

3. $i(0) = -3 = \lim_{x\to 0} i(x)$

$\therefore$ continuous at $x=0$!

$y=x-3$



e. $f(x)=\dfrac{1}{x+2}$

f. $f(x)=\dfrac{1}{x^2+4}$

$f(a)$ Exist → $\lim\limits_{x\to a} f(x)$ Exist → $\boxed{f(a) = \lim\limits_{x\to a} f(x)}$

3. Find the values of each constant that would make this function continuous.

a=? b=? c=?

a.

$$f(x)=\begin{cases} \boxed{-2x+a} & \text{if } x\le -1 \\ \boxed{x^2+b} & \text{if } -1< x\le 2 \\ \dfrac{1}{x}+\cancel{c}\,2a & \text{if } 2\le x \end{cases}$$

At $x=-1$: $\lim\limits_{x\to -1^-} -2x+a = \lim\limits_{x\to -1^+} x^2+b$

Sub $x=1$

$-2(-1)+a = (-1)^2+b$

$2+a = 1+b$

$\boxed{1+a=b}$ ①

At $x=2$: $\lim\limits_{x\to 2^-} x^2+b = \lim\limits_{x\to 2^+} \frac{1}{x}+2a$

Sub $x=2$

$2^2+b = \frac{1}{2}+2a$

$4+b = \frac{1}{2}+2a$

$\boxed{b = 2a-\frac{7}{2}}$

$\frac{1}{2}$ - c

$1+a = 2a-\frac{7}{2}$

$1+\frac{7}{2} = 2a-a$

$\frac{9}{2} = a$

$b = \frac{9}{2}+1 = \frac{11}{2}$

$\therefore a = \frac{9}{2}$

$b = \frac{11}{2}$

b.

$$j(x)=\begin{cases} 2x+a & \text{if } x\le -1 \\ 22 & \text{if } -1< x\le 3 \\ -bx^2+31 & \text{if } 3< x\le 5 \\ \sqrt{cx}+b & \text{if } 5< x \end{cases}$$

At $x=-1$:

$\lim\limits_{x\to -1^-} 2x+a = \lim\limits_{x\to -1^+} 22$

$2(-1)+a = 22$

$a = 24$

At $x=3$:

$\lim\limits_{x\to 3^-} 22 = \lim\limits_{x\to 3^+} -bx^2+31$

$22 = -b(3)^2+31$

$22 = -9b+31$

$9b = 9$

$b=1$

At $x=5$:

$\lim\limits_{x\to 5^-} -bx^2+31 = \lim\limits_{x\to 5^+} \sqrt{cx}+b$

$-25b+31 = \sqrt{5c}+b$

$-25(1)+31 = \sqrt{5c}+1$

$5 = \sqrt{5c}$

$25 = 5c$

$5 = c$

HOMEWORK:

① Journal Unit 5 #4

② Textbook pg. 52 #4ace, 5bcdf, 7, 8, 12, 14, 15, 16 (13 questions)

REVIEW:

↳ 3 conditions for continuity at $x=a$

① $f(a)$ Exist

② $\lim\limits_{x\to a} f(x)$ Exist

③ $f(a) = \lim\limits_{x\to a} f(x)$

↳ 3 TYPES of discontinuity.

① Point

② Jump

③ Infinite