Derivatives of Polynomials

$f(x) = x^2 + 2x + 1$

Develop/Prove some short cut RULES instead of doing 1st principles all the time.

Constant Rule

$f(x) = c$ $\{c \in \mathbb{R}\}$

$\boxed{f'(x) = 0}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x+h) = c$

$f(x) = c$

$= \lim_{h \to 0} \frac{c - c}{h}$

$= \lim_{h \to 0} \frac{0}{h}$

$= \lim_{h \to 0} 0 = 0$

Power Rule

$n \in \mathbb{R}$

$f(x) = x^n$

$f'(x) = n x^{n-1}$

ex: $f(x) = x^3$

$f'(x) = 3x^{3-1}$

$f'(x) = 3x^2$

$y = \sqrt{x} = x^{\frac{1}{2}}$

$y' = \frac{1}{2} x^{\frac{1}{2} - 1}$

$= \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}}$

$= \frac{1}{2\sqrt{x}}$

$\sqrt[3]{\ } = \frac{1}{3}$

$\sqrt[4]{\ } = \frac{1}{4}$

$\sqrt[5]{\ } = \frac{1}{5}$

Constant Multiple Rule

$\{k \in \mathbb{R}\}$

$f(x) = k\, g(x)$

$\boxed{f'(x) = k\, g'(x)}$

ex: $y = \underset{k}{2} \underset{g(x)}{x^3}$

$g(x) = x^3$

$\boxed{g'(x) = 3x^2}$

$y' = 2(3)x^2$

$= 6x^2$

Sum&Difference Rule

$h(x) = f(x) \pm g(x)$

$h'(x) = f'(x) \pm g'(x)$

ex: $y = \underbrace{x^3}_{f(x)} + \underbrace{2x}_{g(x)}$

$f(x) = x^3$

$f'(x) = 3x^2$ +

$g(x) = 2x$

$g'(x) = 2$

$y' = 3x^2 + 2$

$f'(x)$, y' → find the derivative function

1. Differentiate each function. Simplify to create an expanded polynomial first.

a. $y=5x^6-4x^3+6$

$y' = 5(6)x^{6-1} - 4(3)x^{3-1} + 0$

$y' = 30x^5 - 12x^2$

b. $f(x)=-3x^5+8\sqrt{x}-\dfrac{9}{x}$ $8x^{\frac{1}{2}}$ $9x^{-1}$

$f'(x) = -3(5)x^{5-1} + 8\left(\frac{1}{2}\right)x^{\frac{1}{2}-1} - 9(-1)x^{-1-1}$

$= -15x^4 + 4x^{-1/2} + 9x^{-2}$

$= -15x^4 + \dfrac{4}{\sqrt{x}} + \dfrac{9}{x^2}$

Product Rule

c. $g(x)=(2x-3)(x+1)$ $f(x)$, $h(x)$ factored form $f'(x)=2$, $h'(x)=1$

$g'(x) \neq (2-0)(1+0)$

Expand & Simplify First!

$g(x) = 2x^2 - x^1 - 3$

$g'(x) = 2(2)x^{2-1} - 1x^{1-1} - 0$

$= 4x^1 - x^0$

$\boxed{g'(x) = 4x - 1}$

Same

$g'(x) = f'(x)h(x) + f(x)h'(x)$

$= 2(x+1) + (2x-3)(1)$

$= 2x+2+2x-3$

$\boxed{g'(x) = 4x - 1}$

d. $h(x)=\dfrac{-8x^6+8x^2}{4x^5}$

Wrong! Simplify first!

$h'(x) \neq \dfrac{-8(6)x^{6-1} + 8(2)x^{2-1}}{4(5)x^{5-1}}$

$h(x) = \dfrac{\cancel{4}(-2x^6+2x^2)}{\cancel{4}x^5}$

$= (-2x^6+2x^2)(x^{-5})$

$h(x) = -2x^1 + 2x^{-3}$

$h'(x) = -2 + 2(-3)x^{-3-1}$

$= -2 - 6x^{-4}$

$\boxed{h'(x) = -2 - \dfrac{6}{x^4}}$

2. Determine the equation of the tangent to the curve $f(x)=4x^3+3x^2-5$ at $x=-1$, $f(-1) = y = 4(-1)^3+3(-1)^2-5 = -6$

① find slope, m_T, at $x=-1$ ⇒ $f'(-1)$

② plug m_T and (x,y) into $y=mx+b$. Solve for b.

③ $y = m_T x + b$

① $f'(x) = 12x^2 + 6x$

$m_T = f'(-1) = 12(-1)^2 + 6(-1)$

$= 12 - 6$

slope $= m_T = 6$

② $m=6$, $(-1,-6)$

$y = mx + b$

$-6 = 6(-1) + b$

$-6 + 6 = b$

$0 = b$

③ $m = 6$, $b = 0$

$\boxed{y = mx + b \quad y = 6x + 0}$ ∴ $\boxed{y = 6x}$

3. Determine the point(s) (x,y) on the graph of $f(x)$ $y=x^2(x+3)$ where the slope of the tangent is 24. $\boxed{f'(x) = 24}$

$f(x) = x^2(x+3)$

$f(x) = x^3 + 3x^2$

Sub $f'(x) = 24$ $f'(x) = 3x^2 + 6x$

$24 = 3x^2 + 6x$

Solve for x: $0 = 3x^2 + 6x - 24$

→ factor
→ quadratic formula

$0 = 3(x^2 + 2x - 8)$

$0 = 3(x+4)(x-2)$

$x+4=0$ or $x-2=0$

$x_1 = -4$ $x_2 = 2$

$f(-4) = (-4)^2(-4+3) = -16$

$f(2) = 2^2(2+3) = 20$

∴ $(-4,-16)$ and $(2,20)$

Result when multiply.

the product of 2 and 3 is 6

(2)(3) = 6 ← product.

Product Rule

1. Show that the product rule is not just the derivatives of the two factors multiplied.

$h(x) = f(x) \cdot g(x) \Rightarrow h'(x) \neq f'(x) \cdot g'(x)$

2. Develop/Prove the product rule.

Product Rule

$h(x) = f(x)\, g(x)$

$h'(x) = f'(x)\, g(x) + f(x)\, g'(x)$

3. Differentiate

a. $h(x) = \underbrace{(x^2+1)}_{f(x)}\underbrace{(1-x)}_{g(x)}$

$f(x) = x^2+1 \quad g(x) = 1-x$

$f'(x) = 2x \quad g'(x) = -1$

$h'(x) = f'(x)g(x) + f(x)g'(x)$

$= (2x)(1-x) + (x^2+1)(-1)$

$= 2x - 2x^2 - x^2 - 1$

$= -3x^2 + 2x - 1$

b. $(x^3+4x^2-6x+1)(-3x^2+9x-2)$

$f'(x) = 3x^2 + 8x - 6 \quad g'(x) = -6x + 9$

$h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(x) = (3x^2+8x-6)(-3x^2+9x-2) + (x^3+4x^2-6x+1)(-6x+9)$