4. Develop the power of a function rule by looking at the pattern from these two questions

a. $f(x) = g^2(x)$

$f = g^2 \Rightarrow f = g \cdot g$

$f' = g'g + gg'$

$= 2gg'$

$y = (x+2)^2 = (x+2)(x+2)$

$y' = 2(x+2)(1)$

$\boxed{y' = 1(x+2) + (x+2)(1) = 2(x+2)}$

b. $f(x) = g^3(x)$

$f = g \cdot g \cdot g$

$f' = g'gg + gg'g + ggg'$

$= 3ggg'$

$= 3g^2g'$

$y = \boxed{x+2}^3$

$y' = 3(x+2)^{3-1}(1)$

$= 3(x+2)^2$

Power of a Function Rule

Special case of Chain Rule

$f(x) = g^n(x)$ $\quad f = g^n$

$f'(x) = n\, g^{n-1}(x)\, g'(x)$

5. Differentiate

a. $y = \underbrace{(4x^3 - 2x^2 + x - 10)}_{g(x)}^{5}$ $\quad n=5$ $\quad g'(x) =$

$y' = 5(4x^3 - 2x^2 + x - 10)^{5-1}(4x^3 - 2x^2 + x - 10)'$

$y' = 5(4x^3 - 2x^2 + x - 10)^4(12x^2 - 4x + 1)$

b. $y = \sqrt{6 - 5x^3} = (6 - 5x^3)^{\frac{1}{2}}$

$y' = \frac{1}{2}(6 - 5x^3)^{\frac{1}{2}-1}(6 - 5x^3)'$ Not simplify

$y' = \frac{1}{2}(6 - 5x^3)^{-\frac{1}{2}}(0 - 15x^2)$ ← stop here

$y' = \dfrac{-15x^2}{2\sqrt{6 - 5x^3}}$ ← Simplified!

6. Differentiate

Product Rule + Chain Rule

c. $y = \underbrace{(x^3 + 5x)}_{f(x)}\underbrace{(x^2 - 3)^4}_{g(x)}$

$f'(x) = 3x^2 + 5$

$g'(x) = 4(x^2 - 3)^3(2x)$

$y' = f'g + fg'$

$= (3x^2 + 5)(x^2 - 3)^4 + (x^3 + 5x)[4(x^2 - 3)^3(2x)]$ ← Not simplify.

$= (x^2 - 3)^3[(3x^2 + 5)(x^2 - 3) + (x^3 + 5x)(8x)]$

$= (x^2 - 3)^3(3x^4 - 9x^2 + 5x^2 - 15 + 8x^4 + 40x^2)$

$= (x^2 - 3)^3(11x^4 + 36x^2 - 15)$ ← Simplified!

d. $y = \dfrac{5x - 3}{x^2 + 4x} = \underbrace{(5x - 3)}_{f(x)}\underbrace{(x^2 + 4x)^{-1}}_{g(x)}$

$f'(x) = 5$

$g'(x) = -1(x^2 + 4x)^{-2}(2x + 4)$

$y' = f'g + fg'$

$= 5(x^2 + 4x)^{-1} + (5x - 3)(-1)(x^2 + 4x)^{-2}(2x + 4)$ ← Not simplified

$= \dfrac{5}{x^2 + 4x} + \dfrac{(5x - 3)(-2x - 4)}{(x^2 + 4x)^2}$ LCD

$= \dfrac{5x^2 + \cancel{20x} - 10x^2 - \cancel{20x} + 6x + 12}{(x^2 + 4x)^2}$

$= \dfrac{-5x^2 + 6x + 12}{(x^2 + 4x)^2}$ ← Simplified!

Quotient Rule

1. Develop/Prove the quotient rule by using the power rule.

$y = \frac{5x-3}{x^2+4x}$

$f(x) = 5x-3$

$f'(x) = 5$

$g(x) = x^2+4x$

$g'(x) = 2x+4$

$g^2(x) = (x^2+4x)^2$

$y' = \frac{f'g - fg'}{g^2}$

$= \frac{(5)(x^2+4x)-(5x-3)(2x+4)}{(x^2+4x)^2}$

$= \frac{5x^2 + \cancel{20x} - 10x^2 - \cancel{20x} + 6x + 12}{(x^2+4x)^2}$

$= \frac{-5x^2+6x+12}{(x^2+4x)^2}$ ← Simplified!

Quotient Rule $\div$

$\frac{f}{g} = \frac{f'g - fg'}{g^2}$

2 6 $\frac{6}{2}$

$= 2$

2. Differentiate

a. $q(x) = \frac{6x-5}{x^3+4}$

optional:

$f = 6x-5$

$f' = 6$

$g = x^3+4$

$g' = 3x^2$

$g^2 = (x^3+4)^2$

$q'(x) = \frac{f'g - fg'}{g^2}$

$= \frac{(6)(x^3+4)-(6x-5)(3x^2)}{(x^3+4)^2}$ ← Not Simplify

$= \frac{6x^3+24-18x^3+15x^2}{(x^3+4)^2}$

$= \frac{-12x^3+15x^2+24}{(x^3+4)^2}$ ← Simplified!

b. $p(x) = \frac{x+3}{\sqrt{x^2-1}}$

$f = x+3$

$f' = 1$

$g = (x^2-1)^{\frac{1}{2}}$ ← use chain Rule.

$g' = \frac{1}{2}(x^2-1)^{-\frac{1}{2}}(2x)$

$g^2 = (\sqrt{x^2-1})^2 = x^2-1$

$p'(x) = \frac{f'g - fg'}{g^2}$

$= \frac{(1)(x^2-1)^{\frac{1}{2}} - (x+3)(\frac{1}{2})(x^2-1)^{-\frac{1}{2}}(2x)}{x^2-1}$

$= \frac{(x^2-1)^{-\frac{1}{2}}[(x^2-1)^1 - (x+3)(x)]}{x^2-1}$

$= \frac{\cancel{x^2} - 1 - \cancel{x^2} - 3x}{(x^2-1)^{\frac{1}{2}}(x^2-1)^1}$

$= \frac{-3x-1}{(x^2-1)^{3/2}}$ ← Simplified!

$\frac{a^{\frac{1}{2}}}{a^{-\frac{1}{2}}} = a^{\frac{1}{2}-(-\frac{1}{2})}$

$= a^1$

Horizontal tangent at $x=a$ $\Rightarrow m=0=f'(a)=0$

$y=mx+b$ $y=\frac{13x}{9}-\frac{23}{9}$

3. Determine the equation of the tangent to the curve $y=\dfrac{x^2-3}{5-x}$ at $x=2$

① find derivative function

$$y'=\frac{f'g-fg'}{g^2}$$

$$=\frac{(2x)(5-x)-(x^2-3)(-1)}{(5-x)^2}$$

$$=\frac{10x-2x^2+x^2-3}{(5-x)^2}$$

$$=\frac{-x^2+10x-3}{(5-x)^2}$$

② Sub $x=2$ in y' to find slope of tangent.

$$m=\frac{-(2)^2+10(2)-3}{(5-2)^2}=\frac{13}{9}$$

③ Sub $x=2$ into y to find y coordinate

$$y=\frac{2^2-3}{5-2}=\frac{1}{3}$$

④ Sub $m=\frac{13}{9}$ and point $(2,\frac{1}{3})$ in $y=mx+b$ to find b

$$\frac{1}{3}=\frac{13}{9}(2)+b$$

$$\frac{1}{3}=\frac{26}{9}+b$$

$$\frac{1}{3}-\frac{26}{9}=b$$

$$-\frac{23}{9}=b$$

⑤ ∴ Equation of tangent at $x=2$ is

$$y=\frac{13}{9}x-\frac{23}{9}$$

4. Suppose the function $V(t)=\dfrac{50000+6t}{1+0.4t}$ represents the value, in dollars, of a new car t years after it is purchased.

a. What is the Instantaneous rate of change of the value of the car at 2years? 5years? 7years?
b. What is the initial value of the car?
c. Explain how the values in a. can be used to support an argument in favour of purchasing a used car, rather than a new one.

a) ① find derivative function

$$V'(t)=\frac{(6)(1+0.4t)-(50000+6t)(0.4)}{(1+0.4t)^2}$$

$$=\frac{6+\cancel{2.4t}-20000-\cancel{2.4t}}{(1+0.4t)^2}$$

$$=\frac{-19994}{(1+0.4t)^2}$$

② Sub $t=2$

$$V'(2)=\frac{-19994}{[1+(0.4)(2)]^2}=-6170.99 \text{ \$/year}$$ $\Rightarrow$ lose \$6170.99 per year.

$V'(5)=-2221.56$ \$/year

$V'(7)=-1384.63$ \$/year

b) initial $\Rightarrow t=0$

$$V(0)=\frac{50000+6(0)}{1+0.4(0)}=50000 \text{ dollar.}$$

∴ the initial value of the car is \$50000.

c)

Differentiate.

$f(g(h))$

$$y=\left(\left((3x^2+x)^{-1}-5\right)^3+10\right)^6$$

$$y'=6\left(\left((3x^2+x)^{-1}-5\right)^3+10\right)^5(3)\left((3x^2+x)^{-1}-5\right)^2(-1)(3x^2+x)^{-2}(6x+1)$$

Rule:

① Power Rule
② Constant Rule
③ Constant Multiple Rule
④ Sum & Difference Rule
⑤ Product Rule
⑥ Power of function Rule (Chain Rule)
⑦ Quotient Rule

Hand in: ① Journal Unit 6 # 1abc, 2abcd

② Worksheets
- Derivative of Polynomial function
- Product Rule
- Chain Rule
- Quotient Rule.

③ Quizes
- Sketching derivatives
- Derivatives of Polynomial functions
- Product Rule